STARSPOT CYCLES OF RS CANUM VENATICORUM TYPE STARS

BY

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STARSPOT CYCLES OF RS CANUM VENATICORUM TYPE STARS

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The starspot model was used to account for the photometric variations of RS Canum Venaticorum binaries. The twelve light curves of eight epochs for RS CVn, eleven light curves of six epochs for AR Lac and twelve light curves of nine epochs for V711 Tau were analyzed to investigate the cyclic variations of these systems.

The spot parameters, as well as other elements of the systems, have been determined for each epoch by adjusting the parameters to fit the light curves. To avoid subjective parameter adjustment, the Wilson and Devinney differential correction computer program was adopted and modified for the adjustments of the spot parameters.

Models were tested using one or two circular spots on the secondary stars. The adjustable parameters are latitudes, longitudes, radii, and temperatures of the spots, the surface potentials, albedos, temperatures of the two stars and the orbital inclination. The light curve fitting was done by adjusting the spot parameters and other parameters until a good fit to the entire light curve was obtained. We compared the one-spot model and the two-spot model for the three systems. A two spot-model adequately reproduces most light curves for the three systems. Of course, each model spot discussed here probably represents a spot grouping on the real star.

To find the starspot cycles, only the spot longitude variations were considered. For RS CVn, starspot cycles of 9.22 years and 9.00 years were determined for two spots. These cycles agree with the 9.4 year cycle of the distortion-wave found by Catalano et al. For AR Lac, the cycles of the two spots are 11.26 years and 15.18 years, respectively. Finally 7.48 year and 7.09 year cycles were determined for V711 Tau.

CHAPTER I
INTRODUCTION

The study of RS Canum Venaticorum (RS CVn) binaries as a group of stars has been growing enormously in the last decade because they have been recognized as having some unusual properties which set them apart from Algol type binaries.

From a historical point of view, Struve (1946) was probably the first to recognize that there was a group of eclipsing binaries with emission lines. He discussed the existence of bright lines of Ca II in these eclipsing systems where both components were usually later than F5. He suggested that the emission lines belonged to the fainter, cooler and larger component of the binary systems and the formation of the Ca II lines was attributed in part to ordinary resonance scattering, so that the two localized tidal bulges of the chromosphere produced emission lines predominantly when they are both seen from the side, that is, at phase 0.25P and 0.75P.

Hiltner (1947) decided to collect the scattered materials because a number of late-type eclipsing variables had been found with H and K Ca II emission and they formed a definite group of stars. He listed thirteen eclipsing binaries with

H and K Ca II emission. The periods of these binaries are well distributed from 1.98 to 39.28 days and the spectral class of the component with emission varies from F2 to K2.

Gratton (1950) discussed a group of nineteen late-type H and K emission binaries which included Hiltner's thirteen. He investigated a period-luminosity relationship among this group. Bidelman (1954) published a list of stars which were known to show emission lines and whose spectral types were later than B.

Plavec (1967) included five stars (RS CVn, WW Dra, Z Her, AR Lac, and SZ Psc) in his study. He pointed out that each star had a mass ratio near unity, while most Algol-like binaries had mass ratios much smaller than unity. He concluded that, based on their mass ratios, their total system masses and their spectral characteristics the above stars formed a distinct group of eclipsing binary systems.

Popper (1970) presented a list of twenty-two eclipsing binaries which represented the most comprehensive list of possible candidates for the group.

Oliver (1974) was the first to formally propose a set of observational characteristics to define the group called RS Canum Venaticorum Stars.

Hall (1976) reviewed the many observational properties of the RS CVn type binaries. He examined the twenty-four systems considered by Oliver (1974) and added the orbital period range (one day - two weeks) to the working definition

of the group. Thus his proposed definition is as follows: the orbital period is in the range one day to two weeks, the hotter component is of spectral type F or G and luminosity class V or IV, and strong H and K emission is seen outside eclipse. Hall listed the observed properties and physical characteristics of the RS CVn binaries as follows:

1. Orbital period between one day and two weeks.
2. Strong H and K emission seen outside eclipse.
3. Hotter star is F or G, V or IV.
4. H and K emission is from the cooler star (or both).
5. Cooler star is around K0 IV.
6. Hydrogen alpha emission is seen outside eclipse.
7. Wave-like distortion outside eclipse.
8. Wave migration towards decreasing phase.
9. Variable depth of primary minimum.
10. Variable displacement of secondary eclipse.
11. Irregular light curve variations.
12. UV excess in one or both components.
13. IR excess in one or both components.
14. Radio emission.
15. Variable orbital period.
16. Period variations correlated with migration.
17. Mass ratio near unity.
18. Binary is detached.

The first three are exhibited by all twenty-four systems. This is why he proposed the first three as a working definition of RS CVn binaries.

After the realization that RS CVn binaries formed a group of binary systems, one of the most remarkable discoveries pertaining to this group was the detection of strong radio emission from several members. The radio emission has been detected by many observers: Gibson and Hjellming (1974), Gibson et al. (1975), Spangler (1977), Owen et al. (1976), Feldman et al. (1978), to mention a few. Most observers agree that the mechanism of the radio emission is almost certainly nonthermal gyro-synchrotron radiation. X-ray emission was detected from several members of this group. Walter et al. (1978) suggested that at least some, and perhaps all, RS CVn binaries are sources of soft X-rays.

It is our opinion that characteristics No. 7 and 8, the migrating wave-like distortion in the light curve, are the most fascinating aspects of RS CVn binaries. Thus, in this dissertation, we will concentrate on the question of the wave-like distortion. It appeared that the light curves just varied irregularly with a variety of time scales until the astronomers at Catania (Chisari and Lacona 1965; Catalano and Rodono 1967, 1969) observed a quasi-sinusoidal wave in the light curve of RS CVn outside eclipse and they found that the wave migrated slowly toward decreasing orbital phase.

Several models have been proposed to explain the problem of wave-like distortion outside eclipse. Two of these were competing theories. The spot hypothesis, in which a

grouping of cooler, darker regions is distributed unevenly on the surface of on the components of such a system, explains the stellar brightness variations. This hypothesis was first advanced by Kron (1947) to explain the light curve complications of AR Lac. The other competing theory, first advanced by Catalano and Rodono (1967, 1969) was the circumstellar ring model in which a ring of material circled the primary component, whose equatorial plane was inclined with respect to the orbital plane. Accordingly, the continuous variations of the projected area of the disk during orbital revolution would cause the observed wave-like variation of the light curve.

Oliver (1975) has discussed the tilted ring model from the point of view of celestial mechanics and has shown that it is extremely difficult to explain its present existence in a quantitative way. Eaton and Hall (1979) reproduced light curves using different mechanisms (the spot, ring, pulsation) and showed that the spot model gives significantly better fits in detail than any other hypothesis. The spot theory is now widely accepted to explain the photometric complications of RS CVn binaries.

A goal of this dissertation is to investigate the existence of the starspot cycles for RS CVn binaries when we assume that the wave-like distortion is produced by the starspots. For this work, RS CVn, AR Lac, and V711 Tau were selected because enough photoelectric light curves of these

systems have been accumulated to analyze the cyclic, photometric variations for the last decade as compared to other less well observed RS CVn binaries. The Wilson-Devinney (WD) (1971) Differential Correction computer program has been modified to adjust the spot parameters as well as the elements of the systems. Thus, complete elements, including spot parameters, have been determined for each epoch by the method of WD differential correction. Then a series of the solutions for each epoch will make it possible to investigate the spot migration and the spot cycle if it exists. In meeting this goal, new improved elements have been determined for AR Lac, whose previous published elements were determined by the usual rectification method. Also, the first photometric solution for V711 Tau has been carried out.

CHAPTER II
HISTORICAL BACKGROUD OF RS CVN BINARIES

RS Canum Venaticorum

RS Canum Venaticorum was first reported as a variable star by Ceraski (1914). Hoffmeister (1915) produced the first light element using his photographic observation:

Min I = JD Hel 2420297.571 + 4.786960E

Hoffmeister (1919) continued to observe RS CVn and revised his ephemeris. The Mount Wilson annual report for 1923 mentioned that observation of primary eclipse showed the spectral type to be K0 (Wood, 1979). Sitterly (1930) made the first photometric sclutions using both visual and photographic estimates. He noted asymmetries both in primary eclipse and between the eclipses, and a mean difference between two maxima of 0.07 magnitude. He found that the period steadily increased between 1900 and 1922.

In the same year, Joy (1930) classified the components as F4n and dG8. C. Payne-Gaposchkin (1939) made a study of the period changes and light curve asymmetries of RS CVn as a part of her study of variable stars. She collected approximately 4000 photographic estimates of brightness from Harvard plates exposed between the years 1892-1938. She confirmed a period change of the same type as indicated by

Sitterly. With regard to the difference in height between the two maxima, she found that the photographic and visual observations were not in precise agreement, and the photographic observations showed quite definitely the shallow secondary minimum, while Sitterly failed to show a secondary minimum. She estimated that an absolute orbital radius of 2,000,000,000 km would be necessary to explain the period changes as a light-time effect. She concluded that since the difference in the heights of the maxima was apparently greater for visual than for photographic magnitudes, it should be attributed to the fainter and redder star.

Hiltner (1947) observed the H and K lines of Ca II in emission and noted that a group of stars exists which shows Ca II emission. Moreover, the emission approaches zero at the secondary minimum when only a small fraction of the secondary star is eclipsed by the primary component. He concluded from a study of RS CVn and similar eclipsing systems that the Ca II emission was confined to restricted areas of the fainter star such as the ends of the tidal bulges.

Keller and Limber (1951) obtained photoelectric observations without filter. They noted that the maximum following primary minimum was higher than the other maximum, confirming the observations of both Sitterly and C. Payne-Gaposchkin. The amplitude of the difference was about 0.04 magnitude. Photometric elements were calculated by means of

Merrill's tables. The absolute dimension were found with Joy's spectroscopic elements and are tabulated in Table 1.

TABLE 1

Keller and Limber's Results

	Bright Star	Faint Star
Radius (Solar Units)	1.77	4.83
Mass (Solar Units)	1.84	1.70
Luminosity (L1 + L2 = 1)	0.726	0.274
k = 0.365 i = 82.5 + 0.5 degrees		

Popper (1961) obtained spectroscopic and photometric observations which showed that the radii of the components were smaller than previous evaluations. His photometric observations showed the light of the cooler star, as observed during total eclipse, increased by 0.3 magnitude between June 1956 and January 1959, with essentially no change on color and no correspinding increase in the light of the quadratures. The long observed asymmetry of the light curve outside eclipse was found to have the same color as the K-type subgiant. He noted that this persistent asymmetry could be caused by a slight pulsation with the same period as the orbital motion. His results are summarized in Table 2.

Chisari and Lacona (1965) observed RS CVn in 1963 and 1964 using a 30 cm cassegrain telescope with a yellow filter

TABLE 2

Popper's Results

	Bright Star	Faint Star
Radius(a)	0.09-0.12	0.20-0.27
Radius(Solar Units)	1.50-2.00	3.3 -4.6
Mass(Solar Units)	1.32-1.36	1.38-1.42
Luminosity(Solar Units)	3.7 -7.8	5.0 -9.2
B-V	0.39-0.44	0.91
Effective Temp.	6500-6800K	4700K
Bol. Magnitude	2.5 -3.4	2.4 -3.0
Absolute Mag. (V)	2.5 -3.4	2.7 -3.3
Bol.Correction	0.0	-0.3 mag.
m*(sin i)**3	1.32	1.38
a*(sin i)	16.7 solar radii	
i	82 - 90 degrees	
a	16.7 -16.85 solar radii	

whose effective wavelength was 5150A, which is slightly bluer than that for the standard V filter. Their light curves showed very clearly a distortion wave outside eclipse. Observations made during 1964 showed significant changes when compared with those made in 1963, as far as the luminosity outside eclipse is concerned. They concluded that these variation might be attributed to the presence of an absorbing cloud revolving around the system with an independent period. Using Russell and Merrill's method, they found solutions for the 1964 observations. Table 3 contains their solutions.

Catalano and Rodono (1967) reported an additional series of photoelectric observations made in 1965-1966. They made no attempt at a solution, in a classical sence, for the new

TABLE 3

Chisari and Lacona's Results

	Bright Star	Faint Star
Radius (a)	0.112	0.244
Luminosity (L1 + L2 = 1)	0.731	0.268
Period	4.79776 days	
Inclination	84 degrees	
x = 0.8 k = 0.46		

observations, but instead they point out an important fact about the distortion wave in RS CVn. They found that the distortion was shifted in the direction of decreasing phase and completed one cycle in 2400 orbital periods (31.5 years). To explain this phenomenon, a model was developed in which a ring of material circled the primary component whose equatorial plane was inclined with respect to the orbital plane. They suggested that such a ring caused the displacement of the secondary eclipse which had been reported by various observers and attributed to orbital eccentricity. Thus, the observed period changes could not be attributed to apsidal motion. Catalano and Rodono (1969), continuing their observations, added further details. They found the fluctuations were fairly regular outside eclipse, and they could be represented by

$$l = lo + \Delta l/2 \cos(\phi - \theta(t))$$

where ϕ is the phase angle, θ is the phase angle of the minimum of the distortion wave, Δl is the amplitude of the distortion wave, and lo is the luminosity outside eclipse. Unfortunately, for their previous mode, the results were incompatible with the theoretical precession rate of 2400 orbital periods. They found that the distortion wave period was of the order of 800 orbital period (10.5 years) or less.

Hall (1972) proposed a different model which can reproduce the many photometric complications observed in RS CVn and also the period variations. His model describes a region of tremendous sunspot activity darkening the surface of the cool star. This activity is confined to the equatorial region, extending only about 30 degrees in latitude above and below the equator. The region extends only about half way around the star, covering a range of only about 180 degrees in longitude. The activity is periodic, with 1800 orbital cycles between successive maxima. Thus there is a spot cycle of about 23.5 years operating in the cool star of RS CVn. Hall also lists the following complicatons:

1. There is a wave-like distortion which perturbs the light curve outside eclipse.
2. This wave migrates year by year towards decreasing orbital phase.
3. This migration rate is not uniform.

4. Secondary minimum is about half as deep as one would expect from the relative surface brightness of the two stars as deducted from their color indices.
5. The depth of primary minimum varies with time.
6. Secondary minimum does not always occur midway between adjacent primary minima.
7. The amplitude of the wave is not constant.
8. The orbital period changes, despite the fact that both stars are well within their respective Roche lobes.

Hall accounted for the puzzling increase and decrease in orbital period by a continuous ejection of particles from the migrating active region. He calculated an assumed ejection velocity of 3000 km/sec which implies a mass loss of 1/1,000,000 solar mass/year. For the evolutionary status, Hall found that the cool star in RS CVn displayed many of the youthful properties of the T Tauri stars.

Catalano and Rodono (1974) have argued against the existence of the short-term orbital period variations indicated by Hall (1972). They indicated difficulty in determining good times of minima because of asymmetry shown in the minima. They showed that such an asymmetry produced the phase difference between the center of totality and the midpoint between first and fourth contact, so that the times of minima could be correspondingly inaccurate. Therefore they concluded that the observed amplitudes of the short-

term period variations suggested by Hall were not real changes, but those expected because of the above mentioned systematic source of error. Hall (1975) responded to this criticism by calculating corrections for the times of minimum published by Catalano and Rodono (1974). The calculated corrections were small compared to the (O-C) residuals.

Soft X-ray emission from RS CVn was detected by Walter et al. (1978). According to their observations, RS CVn itself appeared significantly more luminous than the other sources seen. They could not distinguish between the possibilities that RS CVn flared for one day or that the quiescent source was much more luminous than the other normal RS CVn binaries.

Ludington (1978) presented photoelectric observations of 1975 and 1976 epochs in the UBV system. His observations were analyzed together with the published Catania data. He substracted a truncated Fourier series representation of the distortion wave from the observed light curve to determine the improved elements by use of Wood's (1971) WINK program. He suggested that since the distortion wave is never flat at maximum in any of the light curves, the spotted region needed to extend at least 180 degrees in longitude on the star, and estimated that the upper limit on the temperature for a spot covering one hemisphere is 4400 K. Eaton and Hall (1979) used synthetic light curves in fitting the light

variation to study the nature of the distortion waves. They found that by using the spot model it was possible to fit both the outside eclipse and eclipse of RS CVn for the 1949 and 1964 epochs. In their model, a lot of small spots rather than a few large ones were put in the restricted ranges of the latitude of the secondary star. Evren et al. (1980) obtained B and V observations of the UBV system at the Ege University Observatory in the 1979 season. The amplitude of the wave was 0.09 magnitude in blue and 0.14 magnitude in yellow light. Catalano et al. (1980) presented the photoelectric light curves carried out at Catania Observatory from 1963 to 1973. Using the phase of the wave minimum, they determined a new period of 9.48 years or 722 orbital periods for the retrograde migration of the wave-like distortion on the light curve. They found that the wave amplitude underwent cyclic changes with a period of 4.74 years. Caton (1981) used a light curve reproducing program developed by Wilson and Devinney (1971) to investigate the starspot parameters using his 1979 RS CVn observations. He found five spots were required for a smooth enough distribution in longitude to reproduce the distortion wave assuming the temperature factor (TF) is 0.9 (TF = the ratio of the spot temperature to the local temperature).

Eaton et al. (1980) indicated a recent sharp acceleration in the migration rate and a simultaneous decrease in the amplitude of the wave using their UBV observations in 1980

season. Blanco et al. (1982) presented a possible scenario in which the migration rate of the photoelectric wave gradually changed its direction, from advancing to receding, provided that the active region was sufficiently long-lived and that it eventually crossed the synchronously rotating latitude. Also, the spotted region would attain maximum angular velocity and gradually decay, as it approaches the equator. As a consequence, the highest receding migration rate of the wave and its lowest amplitude would result.

AR Lacertae

AR Lacertae was discovered to be variable by H. Leavitt in 1907. Because of its period of almost exactly two days, the nature of its variability was not recognized until 1929 by Jacchia (1929), who found AR Lac to be an eclipsing binary. Its period has always been a complicating factor to investigators of the system.

Photoelectric light curves were published by Rugemer (1931), Schneller and Plaut (1932), and Gainullin (1943). Rugemer determined the following ephemeris for AR Lac on the basis of photographic and visual observations made between 1900 and 1931:

$$\text{Min I} = \text{JD Hel } 2426624.378 + 1.983244E$$

Wyse (1934) classified AR Lac spectra as K0 and G5. Wood (1946) obtained 143 photoelectric observations in the blue region and found a solution for the photometric elements. His results are tabulated in Table 4.

TABLE 4

Wood's Results

	Bright Star	Faint Star
Luminosity (L1+L2=1)	0.457	0.543
Radius / (solar unit)	0.194/(1.82)	0.319/(2.99)
Mass (solar unit)	1.40	1.39
Density (solar unit)	0.23	0.05
Inclination: 90 degrees k : 0.609		
Length of Eclipse: 0.3306 day		
Length of Totality: 0.0760 day		

Wood found that Rugemer's elements satisfied the earlier observations but failed completely for his observations. He determined a new ephemeris well-fitted to his observations:

Min I = JD Hel 2426624.3378 + 1.983216E

He was the first to report the intrinsic variation of AR Lac. Because there was wide scatter in secondary minimum, he suggested that the component of greater surface brightness was intrinsically variable.

Kron (1947) made photoelectric observations at Lick in 1938, 1939, 1940, 1946 and 1947. He confirmed an intrinsic variability found by Wood. He indicated that the irregularities repeated themselves with high precision from one revolution of the binary to the next, but that changes in the nature of the irregularities took place within a few weeks to a few months. He explained the peculiar variations with the hypothesis that the surface of the G5 star has upon

it huge dark patches and suggested that the patches covered as much as 20 percent of the projected hemisphere, of which an individual patch covered as much as 3 to 5 percent of the area. This was the first idea for the spot hypothesis.

Harper (1933) has derived orbital elements from his spectrograms. Later Sanford (1951) published new spectroscopic elements which differed but slightly from Harper's. Both results are compared in Table 5.

TABLE 5

Spectroscopic Elements of AR Lacertae

Element	Sanford	Harper
K1	116.1 km/sec	120.0 km/sec
K2	115.6 km/sec	119.2 km/sec
System Velocity	-33.7 km/sec	-36.25 km/sec
a1(sin i)	3,167,000 km	3,263,000 km
a2(sin i)	3,151,000 km	3,247,000 km
m1(sin i)**3	1.29 solar mass	1.39 solar mass
m2(sin i)**3	1.30 solar mass	1.40 solar mass
m1/m2	0.995	0.993

Since the RS CVn binaries as a group of stars have been recognized, AR Lac has been analyzed in detail and observed by many.

Hall et al. (1976) examined 18 light curves, obtained between 1926 and 1974. They found that there was a wave-like distortion outside eclipse which migrated toward decreasing orbital phase at a rate which varies between 10

or 15 yr/cycle and 50 or 60 yr/cycle. The wave amplitude varies between 0.1 and 0.04 magnitude. They indicated that two obvious period decreases (1932 and 1966) and one obvious increase (1957) occurred when the minimum of the wave was at the phase of 0.25 and 0.75, respectively.

Chambliss (1976) obtained photoelectric observations in the UBV system during 1972-1974. For ephemeris and period study, he made separate linear least squares analyses for the data of 1900-32, 1938-40, 1948-59, 1959-65, and 1966-73, because the period of AR Lac had undergone substantial changes. He also calculated photometric elements using the method of Russell and Merrill (1952). The results are tabulated in Table 6. He agreed with the hypothesis that the intrinsic variation arises chiefly from the hotter star because the primary minimum displayed less scatter than did the secondary minimum.

Theokas (1977) analyzed a series of observations made by Kron between 1938 and 1948. His analysis confirmed the period to be nearly constant for sometime and then replaced by a shorter period which itself remains constant. He suggested the nature of the photometric properties might be in a gas stream rather than spot effects.

Srivastava (1981) presented photoelectric light curves in the UBV system observed during the 1975 season. He determined the elements of the system by the method of Russell and Merrill (1952). His elements of the system are tabulated in Table 7.

TABLE 6

Chambliss's Results

Element	V	B	U
x1	0.5	0.6	0.6
x2	0.9	1.0	1.0
k	0.55	0.54	0.55
a1	0.169	0.167	0.170
a2	0.307	0.310	0.309
b1	0.165	0.163	0.166
b2	0.300	0.302	0.302
inclination	86.1	86.9	90.0
L1/(L1+L2)	0.411	0.459	0.552
L2/(L1+L2)	0.589	0.541	0.448
J1/J2	2.30	2.91	4.06

TABLE 7

Srivastava's Results

Element	U	B	V	Mean
x(assumed)	0.8	0.8	0.8	
L1/(L1+L2)	0.555	0.460	0.415	
L2/(L1+L2)	0.445	0.540	0.585	
inclination	75.6	77.1	76.0	76.2
r1	0.176	0.158	0.172	0.169
r2	0.351	0.315	0.343	0.336
J1/J2	5.000	3.407	2.842	3.750

His orbital inclination is smaller than that of Chambliss (1976). Using the colors of the system, he determined the spectral classes of both stars as F8 + G9, which are earlier than those published by Wyse (1934) and Chamliss (1976).

Kurutac et al. (1981) obtained photoelectric observations in two colors, B and V, in 1978 and 1979. They found the orbital period of the system was decreasing by an amount of 14.6 seconds/century. They produced an ephemeris using quadratic least squares as follows:

$$\text{Min I} = \text{JD Hel } 2441593.7115 + 1.98319197E - (0.000004591)*(E*E)$$

The amplitudes of the wave-like distortion in B and V were about 0.09 and 0.12 magnitude in 1978, while being 0.12 and 0.13 magnitude in 1979, respectively. They measured 0.4 period as the amount of shift of the minimum phase of the distortion wave in one year. This was why they suggested that the migration period of the distortion was about 2.5 years. They indicated the cooler KO IV component was responsible for the variable light of the system, because of the variation of the light during totality. This is contradictory to the suggestion made by Wood (1946), Kron (1947) and Chambliss (1976) who have proposed that the hotter component was largely responsible for the intrinsic variability. However the 1979 light curve still remains a question because the light curve obtained in June-July does not match the light curve obtained in September outside of eclipse.

Nha and Kang (1982) presented the photoelectric observations made during 1976 and 1977. They indicated that the depth of the primary minimum showed no variation in any one year but 0.08 magnitude difference between 1976 and 1977. This supports the variation of the cooler K0 component indicated by Kurutac et al. (1981). They tested the spot model and the shell model as an explanation for the cause of the light variability but neither of these satisfied either the color variation during the primary eclipse or the constant color outside eclipse. They deduced the new ephemeris using their new times of the minimum light;

Min I = JD Hel 244093.3081 + 1.9831917E

Ertan et al. (1982) have observed AR Lac and almost completed light curves in two colors, B and V, in 1980 and 1981. They obtained the new times of minimum and indicated that the period of the system was continuing its decrease. Their light curves showed that the brightness decreased from the second to the third contact of the primary eclipse in 1980, while it increased from the second to the third contact in 1981. They found that the wave-like distortion outside eclipses had two maxima. The observation of AR Lac in B and V was continued in 1982 by Evren et al. (1983). They indicated that the system was very active in 1982 like 1979, because when the system was observed in September, it had dimmed about 0.04 magnitude at the phase 0.85 compared to that in June and July, while the brightness at first

maximum seemed to be unchanged. They calculated the decrease in the period, 17.0 + 1.4 seconds/century, using the times of minima obtained photoelectrically between 1961 and 1982.

AR Lac has been investigated in a wide range of wavelength intervals in addition to the optical region. Radio emission from AR Lac was first detected by Hjellming and Blankenship (1973). Gibson and Hjellming (1974) reported a strong radio flare reaching a level of 0.12 Jy at 8085 MHz. At that time they mentioned that AR Lac was the only radio binary besides Algol to attain sufficient flux density levels to allow detailed quantitative interpretation. They found that a number of results had ruled out the possibility of a thermal interpretation of the Algol-type radio events. The radio emssion was also detected by Owen and Spangler (1977), Owen and Gibson (1978), and Doiron and Mutel (1984). No one detected a radio eclipse in AR Lac. To explain the failure to detect a radio eclipse, Owen and Gibson (1978) suggested that the radio emission came from a region characteristic in size of the binary system rather than a region on an individual star. Doiron and Mutel (1984) confirmed a gyro-synchrotron emission mechanism and derived a range of magnetic field strength between 5 and 8 Gauss.

AR Lac was observed in X-rays by Walter et al. (1978, 1980), Swank et al. (1981) and Gibson and Walter (1982). Walter et al. (1983) have made Einstein IPC observations of the X-ray eclipses of AR Lac. Combining IUE and radio

observations, they estimated the structures of coronae of other solar-like stars for the first time. Under the assumption that the X-ray light curve was chiefly attributable to geometric effects, they found that the X-ray luminosities of the G2 and the K0 components comprised about 40 percent and 60 percent, respectively, of the total. They indicated both stars exhibited strong chromospheric activity.

The spectroscopic observations observed by Naftilan and Drake (1977) showed the existence of high-level chromospheric activity in the secondary star, which produced strong H and K emissions. Bopp and Talcott (1978) reported hydrogen alpha observations which showed random variability, while Weiler's (1978) study of hydrogen alpha and Ca II H and K showed both to be nonvariable. Naftilan and Aikman (1981) have investigated the behavior of the Ca II H and K emission lines and found a rapid variability in both the emission strength and line profiles of Ca II H and K. They indicated there was no correlation between the emission intensity and orbital phase of the system, or between the emission strength and the migrating photometric wave phenomena. Nations and Ramsey (1980) observed AR Lac with a CCD detector to study the variability of the hydrogen alpha line. Hydrogen alpha was seen as an absorption feature in both stars with the two components being fairly well resolved. The strength of the absorption was seen to vary

significantly. Hydrogen alpha observations were continued by Huenemoerder and Ramsey (1984). Their observation showed no excess hydrogen alpha emission in AR Lac.

V711 Tauri

V711 Tauri (HR 1099, HD 22468) is the brighter component of the visual double ADS 2644. The light variation of V711 Tauri was first discovered by Cousins (1963), who suspected a variation with an amplitude of Del. V = 0.11 magnitude. This optical variability was confirmed by Landis and Hall (1976), who found a quasi sinusoidal light variation with a period of about 2.822 days. Strong Ca II H and K emission had been discovered in ADS 2644A, V711 Tau, by Wilson (1963), who assigned it a G5V spectral type. It was also known as a radio source by Owen et al. (1976) and as a variable X-ray source by Walter et al. (1978).

Bopp and Fekel (1976) classified this star as a noneclipsing RS CVn binary and determined the spectroscopic period as 2.83782 days and orbital elements given in Table 8. They suggested that temperature and luminosity differences between the primary and the secondary must be small, and the individual types very near G5, because the absorption lines of the two components were nearly equal.

Since it has been known as a RS CVn-type system, V711 Tau has been intensively studied at nearly all wavelength regions.

TABLE 8

Orbital Elements of V711 Tau

P	=	2.83782 + 0.00022 days
Sys. Vel.	=	-14.01 + 0.45 km/sec
e	=	0.027 + 0.014
To	=	JD 2442763.069 + 0.221
K1	=	50.2 + 0.8 km/sec
K2	=	61.6 + 1.5 km/sec
w	=	365 + 27 degrees
a1*(sin i)	=	1.96 + 0.3 km
a2*(sin i)	=	2.41 + 0.6 km
M(Aa)*(sin i)**3	=	0.227 + 0.006 solar mass
M(Ab)*(sin i)**3	=	0.185 + 0.006 solar mass

Bopp et al. (1977) presented photcelectric observations obtained at four different observatories between February 1975 and March 1976. They determined the period of the light variation as 2.837 days. Landis et al. (1978) obtained photoelectric observations at five different observatories during late 1976 and early 1977. The light curves of those showed a wave very similar to the one season in 1975-1976 season. The amplitude in V was unchanged from earlier years and the phase minimum had migrated retrograde 0.074 period since 1976. They suggested an average migration period of 13 years. Photoelectric observations in V and B during January-March 1977 were made by Parthasarathy et al. (1981).

A joint observational campaign on the system V711 Tau was conducted by Weiler (1977) at radio, infrared, optical and ultraviolet frequencies during October 6-18, 1977, with the ultraviolet observations from the COPERNICUS satellite. As a

result of the 1977 campaign, a series of papers was published by many observers. Weiler (1978) obtained ultraviolet observations with the COPERNICUS satellite. These satellite observations were designed to search for Ly alpha and Mg II H and K emission intensity variations over time scales of six hours or more. The observations covered two complete orbital cycles of V711 Tau. He found that the Ly alpha emission strength was marginally correlated with the orbital phase with maximum emission occurring near the phase of photometric wave minimum while minimum emission was generally seen a half-orbital cycle later. This effect may be due to the stellar rotation of asymetrically distributed activity or some undetermined longer term phenomenon.

Batolini et al. (1978) presented UBV, uvby and I photometry during August 1977 - February 1978. They determined the amplitude of the wave (del. V = 0.073 + 0.005) which is smaller than the amplitudes observed around 1963.0, and 1976.0 which were closer to del. V = 0.11 magnitude. The retrograde migration rate, determined by Landis et al. (1978), would have predicted the phase of the minimum light at 0.47 for the epoch of their observations. However, their observations showed the phase of the minimum to be phase 0.67 which is not compatible with Landis and other's prediction. Thus the direction of the migration seems to have changed from retrograde to direct motion. Hearnshaw (1978) obtained 14 spectrograms during the 1977

campaign. He found the hydrogen alpha emission line showed significant profile variations and velocity of the very broad hydrogen alpha emission varied in the phase with the primary star only. Gibson et al. (1978) made radio observations at 4885 and 1465 MHz during the 1977 October campaign. Their result showed that V711 Tau was 20 percent right-hand circularly polarized at 4885 MHz. This is the first significant polarization observed in any of the RS CVn binaries during a quiescent period.

A strong radio outburst was discovered by Feldman et al. (1978). During radio outburst, V711 Tau had been observed at nearly all wavelength regions. Feldman et al. (1978) reported a remarkable series of strong radio outbursts starting on February 20, 1978; the day before, it was an order of magnitude weaker. During the remaining days of February and the first day of March, radio observations were continued along with simultaneous photometry and spectroscopy, both ground-base and satellite. They concluded that the mechanism of radio emission is almost certainly nonthermal gyrosynchrotron radiation in a volume whose characteristic dimension is several times larger than the stellar size(s) but comparable with the binary star separation. Radio observations were also carried out by Aller et al. (1978), Epstein et al. (1978), Furenlid and Young (1978), and Brown and Crane (1978) during the radio outburst of February 1978.

Brown and Crane (1978) supported the conclusion that the radio emission was gyrosynchrotron radiation emitted by energetic electrons that were injected into an ordered magnetic field. Mutel and Weisberg (1978) showed that their polarization observations were inconsistent with a fully relativistic synchrotron process. The consistency of the sense of circular polarization suggested that the emission was associated with a particular magnetic field orientation, perhaps a polar region of one of the stars in the system.

Hydrogen alpha observations were made by Bopp and Talcott (1978) during 1975-1978 and by Fraquelli (1978) during the radio flare of February 1978. The profiles observed during the flare have higher peak intensities and larger equivalent widths than profiles obtained outside the flare at approximately the same orbital phases. Popper (1978) evaluated the intensities of emission lines hydrogen alpha, hydrogen epsilon, and H and K of Ca II in the spectrum at one time during the radio outburst of February 1978. The energy in hydrogen alpha at this was roughly a factor of 2 greater than at the time before the outburst, while the ratio for hydrogen epsilon was a very uncertain factor of 10 greater than a single epoch a year earlier. He also suggested that the spectral classes were K1 IV and G5 IV in order to explain the observed B-V and U-B colors based on UBV photometry by Eggen (1966). Hobbs et al. (1978) suggested the possibility of a direct correlation between

the ratio flare and the chromospheric activities indicated by the H and K lines, using their observations at dispersion of 40 A/mm with an image tube spectrograph.

Chambliss et al. (1978) obtained photolectric V observations during the radio outburst of February 1978. They examined the possible changes in the cptical light curve coincident with the large radio outburst. Except for wave migration, there were no changes in the V light curve. The wave did not vanish and it did not change its shape or amplitude significantly. The mean brightness level did not change by more than 0.005 magnitude. They found the wave migrated from P(min) = 0.67 to 0.74 during 1977.8 - 1978.2, which means direct migartion, while Iandis et al. (1978) reported that the wave was migrating retrograde during 1976.0 - 1977.0.

After the large radio outburst of February 1978, Sarma and Ausekar (1980) obtained the differential magnitude in B and V during November 1978 - February 1979. From the light curves, they obtained average values of 0.9068 for the phase of the minimum and 0.2 magnitude for the full amplitude of the wave. Their observations confirmed Chambliss and other's (1978) conclusion of direct migration of the wave. Photoelectric V observations were continued by Chambliss and Deterline (1979) during February 1979 - March 1979. The total amplitude appeared to be about 0.2 magnitude, which somewhat larger than that reported by Iandis et al. (1978).

The direct migration of the wave minimum was also confirmed. Sarma and Ausekar (1981) continued to observe it in B and V during the 1979-1980 observing season. Their observations yielded a value of 0.972 for the phase of the minimum at 1980.007, thus confirming the nature of direct motion of the migrating wave with a period of about 4.9778 years. The full average amplitude of the wave for the year 1980.007 is 0.123 magnitude.

Guinan et al. (1979) observed this system in a pair of intermediate and narrow band interference filters centered near the rest wavelength of hydrogen alpha line at 6553A. They reported the flare event whose brightness was 0.06 magnitude brighter than normal.

Dorren et al. (1981) have had remarkable success in fitting their 1977-1979 light curves with a relatively simple two spot model in which the two spots are on the more massive member of the binary system, which has been shown to be the chromospherically active star. They determined the spot temperature to the 1800 K cooler than the photosphere, using the observational wavelength dependence of the light curve amplitude in 1977-1978. This model with two circular spots of 26 degree radius at the same latitude (+48 degrees) adequately reproduced the 1977-1978 light curves and the two spots of 31.5 degree radius at latitude +15 degree reproduced the 1979 light curves, respectively.

Spectroscopic observations have been made using the CCD camera at KPNO to test the starspot hypothesis by Ramsey and Nations (1980). They showed that a TiO band system near 8860A strengthened greatly at phase (P=0.81) when starspots have been predicted to be present on the visible hemisphere. This TiO band has been shown to be a sensitive and quantitative temperature indicator in late type stars (Ramsey and Nations 1980).

In recent years (1979-1982), Blanco et al. (1981), Mchin et al. (1982), Olson (1982), Antonopoulow (1983), Rucinski (1983) and Bartolini et al. (1983) obtained photoelectric observations. Blanco et al. (1981) and Bartolini et al. (1983) found that the photometric wave changed from sinusoidal to double-peaked, to almost flat. Bartolini et al. (1983) did obtain photometry on two of the nights during which Guinan et al. (1979) reported their flare. However, a flare-like event was not seen in their observations taken on the same night only 0.33 day earlier. In the framework of the spot model, they guessed the presence on the stellar surface of two spotted areas separated in longitude by about 160 degrees and located at different latitudes due to the two minima per orbital cycle found during 1980-1981.

Because of the great interest in this system, Fekel (1983) improved the orbital elements, using continued spectroscopic observations between 1975 and 1981. New elements are tabulated in Table 9. Fekel reexamined the

possible spectral classifications of V711 Tau. Combinations between G0 V and K2 IV were computed from the spectral classification-color relation tabulated by Johnson (1966). Fekel found that none of the combinations were completely satisfactory because the R and I bandpasses of the RS CVn systems might include a substantial flux contribution from the spotted regions and subgiant colors had to be interpolated between those of dwarfs and giants.

TABLE 9

New Orbital Elements of V711 Tau

P	=	2.83774 + 0.00001 days
To	=	2442763.952
Sys. Vel.	=	15.0 + 0.3 km/sec
e	=	0.0 (assumed)
K(Aa)	=	49.4 + 0.5 km/sec
K(Ab)	=	61.7 + 0.6 km/sec
a(Aa)*(sin i)	=	1.93 + 0.02 km
a(Ab)*(sin i)	=	2.41 + 0.2 km
M(Aa)*(sin i)**3	=	0.224 + 0.005 solar mass
M(Ab)*(sin i)**3	=	0.180 + 0.004 solar mass
M(Aa)/M(Ab)	=	1.25 +0.02

Fraqualli (1984) obtained photographic hydrogen alpha spectrophotometric during both radio-quiescent periods and radio flares during the years 1976-1979. He concluded that a tight coorelation exists between the hydrogen alpha and radio fluxes; the hydrogen alpha equivalent width does not correlate with the distortion wave, and the hydrogen alpha emission profiles are much broader than can be accounted for by rotation and orbital motion.

Doiron and Mutel (1984) made VLBI observations of V711 Tau at 1.65 GHz. The source was found to be unresolved within measurement uncertainties. They derived a upper limit to the source size which was comparable to the overall size of the binary system. The lower limit tc the brightness temperature was 29 billion K for V711 Tau.

CHAPTER III
LIGHT CURVE ANALYSES AND SOLUTIONS

The Wilson and Devinney Program

As a method of analyzing light curves, Wilson and Devinney's (1971) Light Curve program (LC) and Differential Correction program (DC) have been adopted. The actual program is close to that of Wilson (1979). The Light Curve program computes monochromatic light curves from given parameters, with allowance for rotational and tidal distortion, the reflection effect, limb darkening and gravity darkening, while the Differential Correction program solves the inverse problem of finding the parameters from the observations. The model they adopt in the program is the Roche model for close binaries, inscfar as the tidal distortion is concerned.

Each program has eight different modes. MODE 0 is for detached binaries in which the luminosities and temperatures of the components are not coupled. The noncoupling of luminosities and temperatures make this mode similar in that respect to some classic methods (e.g. Russell and Merrill, 1952). MODE 1 and MODE 3 are for contact binaries. The temperature of the secondary star (T2) is computed from the temperature of the primary star (T1) in MODE 1, while T2 may

be adjusted as a free parameter in MODE 3. MODE 2 is for detached binaries with the luminosity of the secondary star (L2) coupled to the temperature of the secondary star (T2) though the Planck function. MODE 4 and MODE 5 are for semi-detached binaries in which the primary fills its limiting lobe in MODE 4, while the secondary fills its lobe in MODE 5. MODE 6 is for double-contact binaries. MODE -1 is for X-ray binaries.

The programs have been updated several times. The most recent version is the eccentric version which includes the spot calculations. However, the Differential Correction program should be modified for this work, because the spot parameters are not adjustable even in the eccentric version. Originally, 22 parameters are adjustable in the DC program. Eight parameters (e : orbital eccentricity, w : longitude of periastron, F1,F2 : ratios of surface rotation rate to synchronous rate for star 1,2, g1,g2 : gravity darkening exponents for star 1,2, A1,A2 : bolometric albedos for stars 1,2) of 22 are changed into the spot parameters. One spot has 4 parameters (i.e. latitude, longitude, radius, temperature) so that parameters of two spots can be now adjusted with other parameters. The derivatives in the modified DC program are tested by running the LC program twice. All parameters in the modified DC program are explained in Table 10.

TABLE 10

Summary of Parameters in Modified DC Program

Curve-Independent Adjustable Parameters	
Co-Lt1, Co-Lt2	Latitude-like coordinates of the spot center 1,2 running from 0 degree at the south pole to 180 degrees at the north pole.
Lg1, Lg2	Longitudes of spot center 1,2, running from 0 degree on the line of central meridian clockwise 360 degrees around the star, as viewed from above.
R1, R2	Radii of the spot 1,2 in degrees. It is the angle subtended by the spot radius at the center of the star.
TF1, TF2	Temperature factors of spot 1,2. It is greater or less than 1.0 as the spot is hotter or cooler than the un-modified local surface temperature.
a	Orbital semimajor axis in solar radii.
V Gam	System velocity.
Phs	phase shift parameter: the phase of conjunction.
i	Orbital inclination of plane of sky.
T1, T2	Mean surface effective temperatures for stars 1,2.
P1, P2	Modified surface potentials for star 1,2.
q	Mass ratio M2/M1, where star 1 is farther from the observer at primary conjunction.
Curve-Dependent Adjustable Parameters	
L1, L2	Relative monochromatic luminosities of stars 1,2 over 4 pi steradians. Because they are 4 pi steradian light outputs, they will be about 4 pi times as large as the computed (output) lights of the model stars, which are per steradian.
x1, x2	Limb darkening coefficients for stars 1,2 in linear cosine law.
L3	Third light in same units are relative flux from the binary system.
Non-Adjustable Parameters	
Period	The orbital period in days.
Long per	The longitude of periastron in degrees.
F1, F2	Ratio of angular rotation rate to the synchronous rate for stars 1,2.
G1, G2	Gravity darkening exponents for stars 1,2.
A1, A2	Bolometric albedos for stars 1,2.
e	The orbital eccentricity.

The principles of operation of the DC program were acomplished by the well known method of differential corrections, using the expression for the total differential of the light values,

$$f(obs) - f(comp) = \frac{\partial f}{\partial P1}\Delta P1 + \frac{\partial f}{\partial P2}\Delta P2 + \ldots + \frac{\partial f}{\partial Pn}\Delta Pn$$

as the equation of condition for a linear least-squares analysis. Here P1, P2, ... Pn are adjustable parameters and f is a function of their parameters. The function f(obs) is understood to be a light value(i.e. relative flux) for photometric observations. Of course $\Delta P1$, $\Delta P2$, ect., are the parameter corrections which will be determined by a linear least-squares and f(comp) is to be computed from the LC program. The derivatives are to be found by

$$\frac{\partial f}{\partial P} = \frac{f(P + \frac{\Delta P}{2}) - f(P - \frac{\Delta P}{2})}{\Delta P}$$

The increments (ΔP) must not be taken too small, for then errors due to inaccuracies in the basic computations of luminosities (L1+L2) become important. Wilson (1983) suggested that for most parameters increments of 0.01 for inclination one degree, and for surface potential and mass

ratio one percent of their values were suitable. In the modified version, increments for spot latitude and longitude are 20 degrees and increments for spot radius and temperature factor are 5 degrees and 0.02 respectively. The DC program permits simultaneous fitting of an arbitrary number of light curves and velocity curves of a given binary. Wilson (1979) described the DC program in detail.

Data Collection

Published light curves for all RS CVn binaries are examined for this work. Unfortunately only three systems (RS CVn, AR Lac, V711 Tau) can be selected in this dissertation, because the study of cyclic variation of the distortion wave requires the accumulation of the light curves. The above three systems have been well observed compared to other systems for 15 or 20 years in the UBV system or a very similar system.

For RS CVn itself, all published light curves and their information are listed in Table 11. It has been continuously observed since its discovery in 1914. Even if many observers in the early period (before 1940's) reported that light curves of RS CVn showed the different levels before and after primary eclipse, their observations were not accurate enough to show a sinusoidal distortion wave outside eclipse of RS CVn. This is why light curves before the 1940's are not adopted in this work. We will not use Oliver's

observations covering only the out-of-eclipse phases, because we need observations covering the whole phase, or at least the eclipse parts, to get better solutions.

TABLE 11

Published Light Curves of RS CVn

Time	Method	Publication	Name
1914	Visual	Hoffmeister (1915)	**
1915	Visual	Hoffmeister (1915)	**
1914-1916	Pg	Maggini (1916)	**
1927-1928	Pg	Schneller (1928)	**
1920-1932	Harvard Plates	Sitterly (1930)	**
1892-1939	Harvard Plates	Payne-Gaposchkin (1939)	**
1949	PE/No Filter	Keller & Limber (1951)	R49
1963	PE/5150A	Chisari & Lacona (1965)	R63
1964	PE/5150A	Chisari & Lacona (1965)	R64
1965	PE/5150A	Catalano & Rodono (1967)	R65
1966	PE/5150A	Catalano & Rodono (1967)	R66
1969	PE/UBV	Oliver (1974)	**
1970	PE/UBV	Oliver (1974)	**
1975	PE/UBV	Ludingtcn (1978)	R75
1976	PE/UBV	Ludington (1978)	R76
1979	PE/UBV	Caton (1981)	R79

**Not used in this work.

For AR Lac, collected light curves are listed in Table 12. Despite being a quite bright star, whole light curves have not been obtained until the late 1970's. Because its period is almost 2 days, it is extremely difficult to cover the whole phase in one observing seascn. However, several observers have attempted to complete a whole light curve in one observing season since it was classified as an RS CVn type binary. The observations for five epochs will be

excluded in this work: the five epochs are the 1938, 1947, 1973, 1974, and 1979. The light curves of the first four epochs covered chiefly the eclipse and the 1979 light curves showed the large difference between beginning (Aug.) and end (Dec.) of the observing season. This difference has not been confirmed as a real change.

TABLE 12

Collected Light Curves of AR Lac

Time	Passband	Publication	Name
1938-1939		Wood (1946)	**
1947	---	Kron (1947)	**
1972	UBV	Chambliss (1976)	A72
1973	UBV	Chambliss (1976)	**
1974	UBV	Chambliss (1976)	**
1975	BV	Srivastava (1981)	A75
1977	BV	Nha & Kang (1982)	A77
1978	UBV	Kurutac et al. (1981)	A78
1979	UBV	Kurutac et al. (1981)	**
1980	BV	Nha et al. (in press)	A80
1981	BV	Nha et al. (in press)	A81

** Not used in this work

V711 Tau has a very short history compared to the other two systems. However, it has been observed by many observers since it was classified as a member of RS CVn binaries. Collected light curves are listed in Table 13. Multi-color observations were selected when more than one light curve is available for the same observing season. Thus observations by Olson (1982) and Flin, and Sztajno (1978)

TABLE 13

Collected Light Curves of V711 Tau

Time	Passband	Publication	Name
Jan 63 - Feb 63	V	Cousins (1963)	T63
Feb 75 - Mar 76	V	Bopp et al. (1977)	T75
Sep 76 - Mar 77	V	Landis et al. (1978)	T76
Jan 77 - Mar 77	BV	Parthasarathy et al. (1981)	**
Aug 77 - Feb 78	V	Batolini et al. (1978)	T77
Oct 77	V	Flin & Sztajno (1978)	**
Aug 77 - Mar 78	UBVI	Olson (1982)	**
Feb 78 - Mar 78	V	Chambliss et al. (1978)	T78B
Nov 78 - Feb 79	BV	Sarma & Ausekar (1980)	T78E
Feb 79 - Mar 78	V	Chambliss & Deterline (1979)	T79B
Nov 79 - Feb 80	BV	Sarma and Ausekar (1981)	T79E
Oct 81	UBV	Olson (1982)	**
Oct 81 - Jan 82	UBV	Rucinski (1983)	T80

** Not used in this work

are not included. Light curves by Parthasarathy et al. (1981) and Olson (1982) are not adopted due to their observational scatters.

In order to satisfy the format of the DC program, magnitude differnces of collected data are converted into normalized relative flux as

$$L = 1/Lo \ (10^{-0.4\ \Delta m})$$

where Lo is the maximum light level and Δm is a magnitude difference. Orbital phases are computed using each ephemeris in Table 14.

TABLE 14

Ephemerides Used for Computing Orbital Phase

Star	Epoch	Period	Reference
RS CVn	2441825.542	4.797855	Ludington (1978)
AR Lac	2440933.3081	1.9831917	Nha & Kang (1982)
V711 Tau	2442766.069	2.83782	Bopp & Fekel (1976)

As the last step of data collection, normal points were made to save computing time. The weight of each point represents the number of observational points.

General Fitting Procedure

There were many computer runs using LC and DC programs before final solutions were selected. We can summarize the fitting procedure as follows:

1. Select initial parameters from published elements.
2. Run the LC program to reproduce the light curve using initial parameters without a spot.
3. Compare the outside eclipse of the light curve reproduced by the LC program to the observations.
4. Adjust the light level (L1) roughly by trial and error.
5. Repeat (3) and (4) several times until the outside eclipse of the reproduced light curve is fitted to the maximum level of the observations.

6. Put the spot on the secondary star which is on the line of sight at the phase minimum where the spot activity shows maximum.
7. Run the DC program with the initial values and adjusted light level (L1) to adjust all parameters by the method of the differential corrections.
8. Repeat running the DC program until a good fit to the entire light curve is obtained.
9. When the fit to the light curve is not good enough, put the second spot on the secondary star and repeat (8).

A Solution of RS CVn

Before running DC program for all epochs, the 1975 epoch was selected as a sample epoch because this epoch is well covered the whole phase in multi-color passbands. For this sample epoch, Caton's (1981) parameters were adopted as initial values, expect for limb darkening coefficients. Table 15 shows adopted values for the sample epoch.

The general fitting procedure was applied for the sample epoch with the one-spot model first. We assumed one circular spot on the secondary star; the longitude of the spot is on the line of sight at the phase minimum where spot visibility is maximum. The co-latitude, radius and temperature factor of the spot were assumed to be 90 degrees(at equator), 20 degrees and 0.75, respectively, for the initial spot

TABLE 15

Initial Input Parameters of RS CVn

Period	4.7948 days	Eccentricity	0.0
Lng Per	90.0 degress	S-M Axis	16.77 Ro
F1	1.0	F2	1.0
G1	0.30	G2	0.30
T1	6700. K	T2	4600. K
Alb1	0.30	Alb2	0.30
Pot1	9.73	Pot2	5.23
V Gam	-890. km/sec	Incl	85.49 degrees
L1v/(L1+L2) v	0.45	L2v/(L1+L2) v	0.45
L2b/(L1+L2) b	0.55	L2b/(L1+L2) b	0.55
x1v	0.65	x2v	0.76
x1b	0.70	x2b	0.90
M2/M1	1.04	L3	0.0

parameters. Longitude, radius, and temperature of the spot, phase shift, inclination, temperature of each star, and light levels of both passbands were adjusted. We did not adjust all of the parameters together, but rather a combination of two or three parameters in a subset. As seen in Figure 1, the DC program improves a fit step by step. The fit was substantially improved until the third running of the DC program. However, it was very difficult to improve the fit after the third running. A summary of fitting procedures for the one-spot model is tabulated in Table 16. As seen in Figure 1, light curves of the last fit are still flat at the phase 0.7-0.9 while observations yield curves descending in the direction of increasing phase. This means that a model needs a much wider spot distribution in longitude or more than one spot(i.e. two spots). Because we

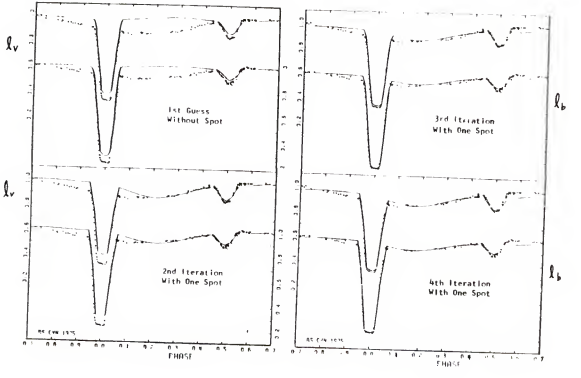

Figure 1: The Fitting Procedure for the One-spot Model, 1975 Epoch, RS CVn.

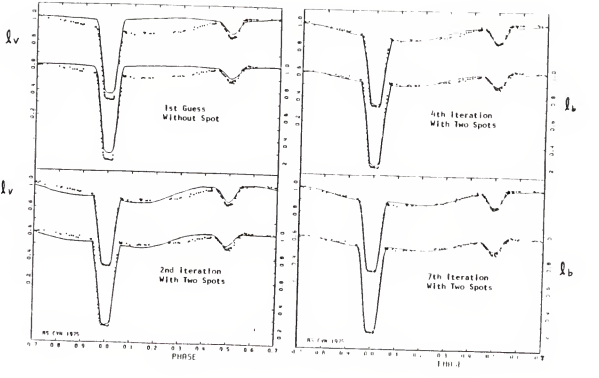

Figure 2: The Fitting Procedure for the Two-spot Model, 1975 Epoch, RS CVn

tried a simple spot model, we chose the two-spot model rather than wider spot distribution in longitude. Of course, each model spot discussed here probably represents a spot grouping on the real star.

TABLE 16

Parameter Adjustment for the One-spot Model (epoch=1975)

Iteration	1	2	3	4
Co-LT1	090.00	090.00	090.00	090.00
LG1	275.00	260.15	247.62	247.62
R1	30.00	31.50	29.20	29.20
TF1	0.75	0.78	0.78	0.72
PS	0.0	0.0	0.0	0.0
INCL	85.552	85.552	86.121	86.121
T1	6810.0	6810.0	7170.0	7170.0
T2	4600.0	4600.0	4600.0	4600.0
L1	7.1100	7.1100	7.4970	7.4860
L2	8.6290	8.6290	9.1130	9.0730
P1	9.3700	9.7300	9.7300	9.7300
P2	5.2300	5.2300	5.2300	5.2300
D	0.3022	0.0483	0.0373	0.0373

Using the two-spot model, the light curve fitting was done again by adjusting a combination of the two-spot's parameters and other parameters in a subset. The quality of a fit was quantified by calculating the sum of the squares of weighted residuals between the observations and computed values (O-C). We also compared the computed and observed light curves graphically. Iterations were repeated until a good fit to the entire light curve was obtained.

Because this is a sample epoch, the albedc of each star was adjusted by the original DC program, not by the modified DC program, with the spot parameters fixed (albedo can nct be adjusted in the modified DC program). The summary cf iterations for the two spot model fcr the sample epoch (1975) is tabulated in Table 17. As seen in Figure 2, the quality of the fit for the two-spot model is much tetter than for the one-spct model for the 1975 epoch.

TABLE 17

Prameter Adjustments for The twc-spct Model (epoch=1975)

Iteration	1	2	3	4	5	6
Co-LT1	090.00	090.00	090.00	090.00	090.00	090.00
LG1	126.00	160.17	160.17	160.17	163.30	163.30
R1	25.00	25.00	25.00	25.00	25.00	25.00
TF1	0.78	0.94	0.85	0.76	0.76	0.81
Co-LT2	090.00	090.00	090.00	090.00	090.00	090.
LG2	234.00	244.35	244.35	255.35	255.35	260.15
R2	38.00	34.00	31.50	31.50	31.50	31.50
TF2	0.82	0.82	0.82	0.82	0.82	0.78
PS	0.0	0.0	0.0	0.0	0.0	0.0
INCl	85.34	85.34	85.34	85.34	85.49	85.55
T1	6700.0	6700.0	6700.0	6700.0	6700.0	6810.0
T2	4600.0	4600.0	4600.0	4600.0	4600.0	4600.0
L1	6.9970	6.8430	6.8430	6.9100	6.9250	7.1100
L2	8.4664	8.3494	9.3494	8.4000	8.4000	8.6290
P1	9.7300	9.7300	9.7300	9.7300	9.7300	9.7300
P2	5.2300	5.2300	5.2300	5.2300	5.2300	5.2300
D	0.0506	0.0745	0.0232	0.0165	0.0159	0.0112

To choose the proper model (the one-spot cr the twc-spot model) for the RS CVn, both models were applied again fcr

the 1965 epoch which was observed in a single-color. Thirteen iterations were done for each model. The parameter evolutions of the one-spot model and the two-spot model are plotted in Figure 3 and Figure 4, respectively. The summaries of iterations for the one-spot and the two-spot models are tabulated in Table 18 and 19, respectively. We obtained the same results as the 1975 epoch: the two-spot model showed a much better fit than the one-spot model. Thus the result of the two sample epochs led us to adopt the two-spot model for all epochs of RS CVn binaries.

After the final solutions of the sample epochs (1975 and 1965) were determined, several adjustable parameters were fixed for other epochs: inclination, potential of each star, and albedo of each star. These parameters are not functions of the spot migrations.

For the adjustment of the spot parameters, longitude, radius, and temperature of the spot have been adjusted first and the spot latitudes have been fixed at the equator until the 6th or 9th iteration. When the spot longitude adjustment was almost finished (conserved), the spot latitude adjustment was begun, because the change of the spot latitude does not affect the light curve variation too much.

We adjusted the temperature of the primary star instead of the secondary star, because the temperature of the secondary star was determined from the color index and the spectral type at the totality. The range of the temperature

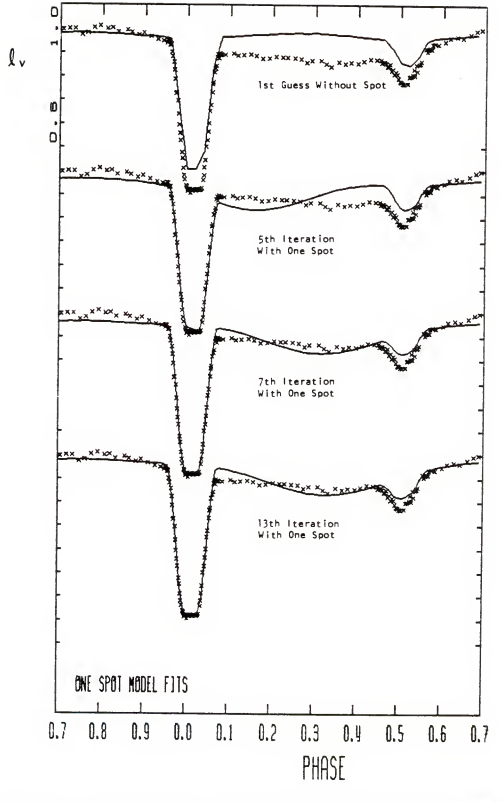

Figure 3: The Fitting Procedure for the One-spot model, 1965 Epoch, RS CVn.

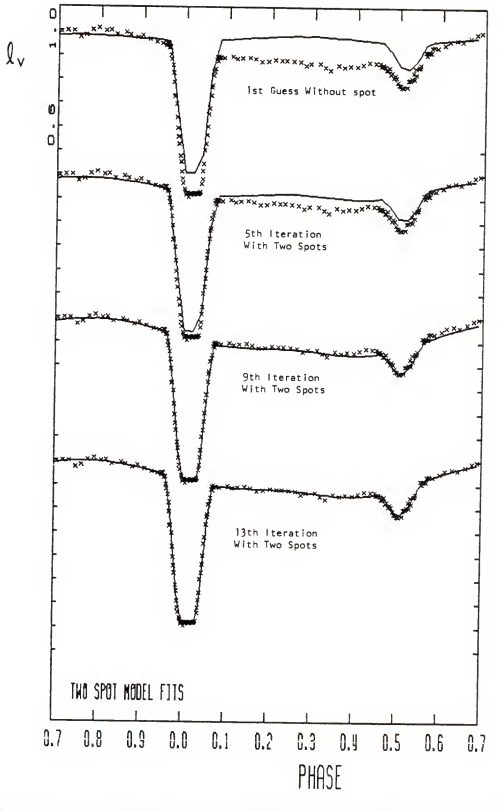

Figure 4: The Fitting Procedure for the Two-spot Model, 1965 Epoch, RS CVn.

TABLE 18

Parameter Adjustments for the One-spot Model (epoch=1965).

Iteration	5	7	9	11	12	13
Co-LT1	60.00	60.00	50.25	50.25	50.25	50.25
LG1	280.21	286.72	289.00	291.05	290.44	291.80
R1	29.78	36.18	39.64	41.67	41.67	41.67
TF1	0.600	0.600	0.602	0.602	0.602	0.602
ps	0.015	0.015	0.015	0.015	0.015	0.015
INCL	85.55	85.55	85.55	85.55	85.55	85.55
T2	4530	4410	4410	4410	4410	4410
P1	9.73	9.73	9.73	9.73	9.73	9.73
P2	5.24	5.24	5.24	5.24	5.24	5.24
L1	7.233	7.903	8.031	7.977	7.874	7.995
L2	Coupled					
D	0.193	0.098	0.104	0.081	0.074	0.067

TABLE 19

Parameter Adjustments for the Two-spot Model (epoch=1965).

Iteration	5	7	9	11	12	13
Co-LT1	90.00	90.00	50.00	46.50	46.50	46.50
LG1	212.98	206.70	210.42	210.42	210.42	210.42
R1	19.92	21.31	29.36	29.83	29.83	30.11
TF1	0.630	0.558	0.558	0.600	0.600	0.600
Co-LT1	90.00	90.00	41.93	50.37	50.37	48.52
LG1	309.97	313.70	313.70	313.70	313.70	313.70
R1	34.53	34.53	55.20	49.36	49.36	51.09
TF2	0.780	0.700	0.848	0.848	0.853	0.853
PS	0.014	0.015	0.015	0.015	0.015	0.015
INCL	85.55	85.55	85.55	85.55	85.55	85.55
T1	6810	6990	6990	6990	6990	6990
T2	4600	4600	4600	4600	4600	4600
P1	9.73	9.73	9.73	9.73	9.73	9.73
P2	5.23	5.23	5.23	5.23	5.24	5.24
L1	7.591	7.370	7.403	7.414	7.414	7.437
D	0.0309	0.0350	0.0217	0.0110	0.0105	0.0103

of the primary star for all epochs is between 6800K and 7000K, except for the 1949 epoch. For the 1949 epoch, the temperature of the primary star was conserved to 7650K, which is higher than those of other epochs. Such a high temperature is caused by the passband used in the DC program. The observations in this epoch were made without a filter so that we assumed a blue passband (4300A).

The epochs between 1949 and 1966 are the single color observations, while the epochs between 1975 and 1979 are the multi-color observations. Eleven to seventeen iterations were done for the solutions for the single color observations and eighteen to twenty iterations for the solutions of the multi-color observations. Even if fewer iterations were done for the single color observations, fits for the single-color observations are better than for the multi-color observations. However, as seen in Figure 5, a two-spot model adequately reproduces most light curves of RS CVn binaries. Final solutions of 8 different epochs are tabulated in Table 20 and stellar radii computed from input data of DC program are tabulated in Table 21.

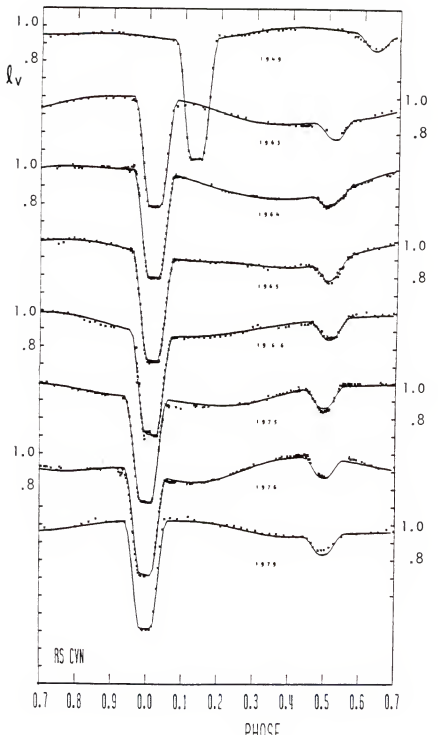

Figure 5: The Fit to the Light Curves of RS CVn.

TABLE 20

Final Solutions of RS CVn for Eight Different Epochs.

Element	1949	1963	1964	1965	1966
Co-LT1	37.57	51.91	74.20	46.50	30.16
LG1	47.63	288.99	267.43	210.42	200.55
R1	29.08	38.46	30.35	30.11	47.57
TF1	0.739	0.543	0.692	0.600	0.728
Co-LT2	47.02	50.00	82.10	48.52	58.18
LG2	162.77	27.91	341.80	313.70	275.13
R2	43.77	43.78	33.34	51.09	33.82
TF2	0.898	0.854	0.828	0.853	0.891
PS	0.133	0.019	0.017	0.015	0.012
INCL	85.55	85.55	85.55	85.55	85.55
T1	7650	7070	6950	6990	6810
T2	4600	4600	4600	4600	4600
P1	9.73	9.73	9.73	9.73	9.75
P2	5.23	5.23	5.24	5.24	5.23
L1v/(L1+L2) v	0.6689	0.6023	0.5884	0.5933	0.5672
L2v/(L1+L2) v	0.3311	0.3977	0.4116	0.4067	0.4378

Element	1975	1976	1979
Co-LT1	71.36	62.73	51.43
LG1	167.10	114.69	318.88
R1	25.00	28.77	30.53
TF1	0.810	0.749	0.691
Co-LT2	70.92	76.56	62.85
LG2	261.87	231.58	69.37
R2	31.50	32.19	23.12
TF2	0.780	0.657	0.663
PS	0.000	-0.002	-0.001
INCL	85.55	85.55	85.55
T1	6810	6840	6800
T2	4600	4600	4600
P1	9.66	9.73	9.73
P2	5.23	5.23	5.23
L1v/(L1+L2) v	0.5734	0.5715	0.5779
L2v/(L1+L2) v	0.4266	0.4285	0.4221
L1b/(L1+L2) b	0.6859	0.6873	0.6812
L2b/(L1+L2) b	0.3141	0.3127	0.3188
L1u/(L1+L2) u			0.7792
L2u/(L1+L2) u			0.2208

TABLE 21

Auxiliary Parameters of RS CVn for the 1975 epoch.

Radius	Star 1	Star 2
Pole	0.1159	0.2421
Point	0.1164	0.2526
Side	0.1160	0.2456
Back	0.1163	0.2503

A Solution of AR Lacertae

The 1981 epoch was selected as a sample epoch. Semi-major axis, longitude of periastron, temperature of each star, limb darkenings, and inclination of the system were adopted from Cambliss's (1976) solution as initial parameters. Mass ratio and the system velocity were adopted from Sanford's (1951) spectroscopic solution. Initial parameters for the 1981 sample epoch are in Table 22.

The initial spot parameters, latitude, radius, and temperature factor were assumed to be 70 degrees, 20 degrees, and 0.9 respectively. This initial spot is smaller and warmer than that of RS CVn itself, because the amplitude of the distortion wave of AR Lac is smaller than that of RS CVn. The longitude of the spot is assumed using the same method of RS CVn. The general fitting procedure was also applied to run the DC program for AR Lac.

A one-spot model and a two-spot model were applied for the sample epoch. Both the model fits were compared

TABLE 22

Initial Input Parameters of AR Lac.

Period	1.9831 days	Eccentricity	0.0
Lng Per	87.5 degrees	S-M Axis	9.22 Ro
F1	1.0	F2	1.0
G1	0.30	G2	0.30
T1	5600 K	T2	4600 K
Alb1	1.00	Alb2	1.00
Pot1	7.0713	Pot2	4.3720
V Gam	-33.7 km/sec	Incl	87.0 degrees
L1v/(L1+L2)v	0.450	L2v/(L1+L2)v	0.450
L1b/(L1+L2)b	0.550	L2b/(L1+L2)b	0.550
x1v	0.52	x2v	0.90
x1b	0.60	x2b	1.00
M2/M1	1.00	L3	0.0

graphically. Unlike RS CVn, there was no big difference between the one-spot model fit and the two-spot model fit for this 1981 sample epoch. When the sum of the squares of weighted residuals is considered, the two-spot model gives a little better fit than the one-spot model.

Both models were applied for all epochs of AR Lac except the 1972 epoch, because it was not clear which model gives a better fit for the sample epoch. The spot parameters, light level (L), phase shift, temperature and potential of each star, were adjusted for all epochs, while inclination and limb darkening coefficients were fixed.

The light curves of the 1981, 1980, and 1978 epochs cover whole phase, while the phase coverages of the light curves for 1977 and 1972 epochs are incomplete. Thus the parameter adjustment was concentrated in the former light curves

rather than the latter light curves. For the 1980 epoch, only the v light curve was analyzed, because the blue light curve observed at different observatories showed a level shifted between the observatories. Spot latitudes for most epochs were not adjusted. As mentioned in a previous section, the derivative with respect to the spot latitude is so small for the warm or small spot near the equator that the derivative may give an unrealistic correction.

The qualities of fits for both models were compared by calculating the sum of the squares of weighted residuals (O-C) tabulated in Table 23. As can be seen in Table 23, the two-spot model fits are better than the one-spot model fits for all epochs.

TABLE 23

Quality of fits for the one-spot and the two-spot model.

Epoch	Sum of Squares of Weighted Residuals	
	One-spot model	Two-spot Model
1981	0.0242	0.0218
1980	Not Applied	0.0070
1978	0.0240	0.0189
1977	0.0206	0.0198
1975	0.0459	0.0290
1972	0.0064	Not Applied

Wood (1946), Chambliss (1976), and Srivastava (1981) have determined a solution for AR Lac, using the usual rectification method. It is interesting to compare the

results of this dissertation with previous by the method of differential correction. A size and a luminosity of each star for the two-spot model are compared with previous solutions. Table 24 shows elements of previous solutions and this dissertation for AR Lac. The 1972 epoch and 1975 epoch were used to determine a solution by Chambliss and Srivastava respectively. Wood and Srivastava adopted a spherical model for their solutions, Chambliss adopted an elliptical model, and a Roche model was used in this dissertation. The two radii of each star in Chambliss' result in Table 24 represent a semi-major axis and semi-minor axis respectively. A size of the star in the DC program is expressed by distance from the center to the pole, to the point, to the side, and to the back. As can be seen in Table 24, the sizes of stars determined in this dissertation are in good agreement with those of Chambliss and in fair agreement with those of Wood and Srivastava. However, the luminosity ratios are different from those of previous solutions. This difference is caused by the spots. The luminosities in this dissertation were computed from unspotted stars.

The observational material of this dissertation is far more extensive and has less scatter than that used by Wood, by Chambliss, and by Srivastava. The method to find a solution included an intrinsic variation of AR Lac. One should expect the elements, determined from the spot model,

TABLE 24

Comparison with Previous Solutions of AR Lac.

	Wood		Chambliss		Srivastava	
PassB and	B	V	B	V	B	
L1	0.457	0.411	0.459	0.415	0.460	
L2	0.543	0.589	0.541	0.585	0.540	
r1	0.194	0.169	0.167	0.172	0.158	
		0.165	0.163			
r2	0.319	0.307	0.310	0.343	0.315	
		0.300	0.302			

	This Dissertation					
Epoch	1972	1975	1977	1978	1980	1981
L1v	0.383	0.385	0.400	0.398	0.420	0.389
L2v	0.617	0.615	0.600	0.602	0.580	0.611
L1b	0.434	0.434	0.459	0.453		0.441
L2b	0.566	0.566	0.541	0.547		0.599
Radius 1						
Pole	0.164	0.163	0.163	0.164	0.164	0.164
Point	0.166	0.166	0.165	0.166	0.166	0.166
Side	0.165	0.164	0.164	0.165	0.165	0.165
Back	0.166	0.165	0.165	0.166	0.166	0.166
Radius 2						
Pole	0.299	0.293	0.300	0.298	0.300	0.295
Point	0.330	0.321	0.331	0.328	0.332	0.323
Side	0.307	0.301	0.309	0.306	0.309	0.303
Back	0.320	0.312	0.322	0.319	0.322	0.315

to be more reliable than those previously published. Final solutions of the two-spot model for 6 different epochs are tabulated in Table 25.

TABLE 25

Final Solution of AR Lac for Six Different Epochs.

Epoch	1972	1975	1977	1978	1980	1981
Co-Lt1	70.00	70.0	70.00	66.09	70.00	67.41
LG1	168.07	65.01	35.64	355.23	268.97	247.35
R1	10.88	15.96	18.28	13.65	17.02	16.91
TF1	0.880	0.691	0.827	0.709	0.9249	0.792
Co-Lt2		70.00	70.00	70.00	70.00	70.00
LG2		284.00	297.74	281.14	142.32	186.86
R2		15.00	14.78	18.41	15.94	9.70
TF2		0.732	0.664	0.832	0.846	0.933
PS	0.001	0.999	0.999	0.998	0.995	0.991
T1	5400	5680	5560	5551	5569	5368
T2	4567	4800	4594	4630	4550	4550
P1	7.0713	7.1013	7.1153	7.0713	7.0713	7.0713
P2	4.3056	4.3730	4.2920	4.3144	4.2882	4.3532
L1v/(L1+L2) v	0.3826	0.3846	0.4004	0.3977	0.4199	0.3894
L2v/(L1+L2) v	0.6174	0.6154	0.5996	0.6023	0.5801	0.6106
L1b/(L1+L2) b	0.4337	0.4339	0.4587	0.4531		0.4406
L2b/(L1+L2) b	0.5663	0.5661	0.5413	0.5469		0.5594

A Solution of V711 Tau

As mentioned in chapter 2, V711 Tau is the primary of the physically associated visual binary ADS 2644, in addition to being a short period spectroscopic binary. To avoid confusion, let us define that the primary (star 1) refers to the hotter, less massive, spectroscopic secondary, G type star, while the secondary (star 2) refers to the cooler, more massive, more chromospherically active K type star.

No photometric solution has been published for V711 Tau. Only a spectroscopic solution was used for initial values of DC program. From Fekel's (1983) results, the inclination,

semi-major axis, system velocity and mass ratio were adopted. The temperature of each star was adopted from Vogt's suggestion, 5600K, and 4750K respectively, obtained from the spectral type and color index (Fekel 1983). The potential of each star was found from the mass ratio table. Albedos were assumed as 1.0 and limb darkening coefficients were adopted from Gingerich (1969). Initial parameters are tabulated in Table 26.

TABLE 26

Input Parameters for V711 Tau.

Period	2.8378	Eccentricity	0.0
Lng. Per	90.0	S-M Axis	11.44 Ro
F1	1.0	F2	1.0
G1	0.30	G2	0.30
T1	5600K	T2	4750K
Alb1	1.00	Alb2	1.00
Pot1	5.100	Pot2	4.250
V Gam	-140 km	Incl	35.0
L1v/(L1+L2)v	0.480	L2v/(L1+L2)v	0.480
L1b/(L1+L2)b	0.520	L2b/(L1+L2)b	0.520
x1v	0.650	x2v	0.760
x1b	0.700	x2b	0.900
M2/M1	1.250	L3	0.000

Because V711 Tau is a non-eclipsing binary, it is better to reduce the number of free parameters by as many as possible. Thus, elements determined by spectroscopy were not adjusted in photometrically for all epochs, even if it is possible to adjust them in the DC program. According to the general fitting procedure, the one-spot model and the two-

spot model were applied for two sample epochs (1978B, 1978E). In those sample epochs, the spot parameters (except latitude), the potential of the secondary star and the light level were adjusted, while the temperature of each star, the potential of the primary star, the inclination of the system, and the limb darkening coefficients were fixed. The results of the sample epoch show no large difference between the one-spot model and the two-spot model. This is why we applied both models for all other epochs. For all other epochs the potential of the secondary star was adopted the value determined from the sample epoch.

The qualities of fits for both model were compared by calculating the sum of the squares of weighted residuals. Table 27 shows a goodness of fit of each model. As can be seen in Table 27, the two-spot model gives better results than the one-spot model for all epochs except the 1978E epoch.

From the sample epochs (1978E, 1978B), stellar radii and luminosities of the components (i.e. L1+L2=1.00) were determined from DC input data and tabulated in Table 28.

Fekel (1983) determined radii of the components, 0.144 for star 1 and 0.322 for star 2, assuming an inclination of 33 degrees and equtorial velocities of 23 km/sec for star 1 and 68 km/sec for star 2. Our radius of star 1 is twice as large as Fekel's radius for star 1. However, Fekel did not describe the method to find the radii of the stars. Final

TABLE 27

Goodness of Fit of V711 Tau for Each Model.

	Sum of Squares of Weighted Residuas	
Model	One-spot Model	Two-spot Model
1963	0.00035	0.00006
1975	0.00057	0.00038
1976	0.00132	0.00121
1977	0.00067	0.00047
1978b	0.00034	0.00086
1978e	0.00094	0.00085
1979b	0.00260	0.00094
1981	0.00047	0.00029

TABLE 28

Stellar Radii and Luminosities for V711 Tau.

	Star 1	Star 2
Luminosity	0.525	0.475
Radius Pole	0.257	0.353
Radius Point	0.275	0.418
Radius Side	0.262	0.368
Radius Back	0.271	0.390

solutions of all epochs for V711 Tau are tabulated in Table 29.

TABLE 29

Final Solutions of V711 Tau for 9 different Epochs.

Epoch	1963	1975	1976	1977
Co-Lt1	55.00	55.00	55.00	55.00
Lg1	68.00	72.03	44.94	59.87
R1	37.13	25.10	27.77	25.21
TF1	0.755	0.745	0.817	0.768
Co-Lt2	55.00	55.00	55.00	55.00
LG2	288.00	345.00	335.14	306.00
R2	27.75	20.68	21.35	11.50
TF2	0.815	0.758	0.742	0.850
Pot1	5.100	5.100	5.100	5.100
Pot 2	4.362	4.349	4.362	4.362
L1v/(L1+L2) v	0.5250	0.5223	0.5250	0.5250
L2v/(L1+L2) v	0.4750	0.4777	0.4750	0.4750

Epoch	1978B	1978E	1979B	1979E	1981
Co-Lt1	55.00	55.00	50.14	55.00	55.00
LG1	66.20	94.16	117.44	129.45	250.91
R1	33.00	34.05	34.30	26.83	23.79
TF1	0.891	0.741	0.722	0.683	0.829
Co-Lt2		55.00	55.00	55.00	55.00
LG2		183.65	195.58	236.83	340.57
R2		28.87	20.98	28.73	20.22
TF2		0.770	0.805	0.882	0.820
Pot1	5.100	5.100	5.100	5.100	5.100
Pot2	4.362	4.296	4.362	4.362	4.362
L1v/(L1+L2) v	0.5006	0.5109	0.5250	0.5250	0.5250
L2v/(L1+L2) v	0.4994	0.4891	0.4750	0.4750	0.4750
L1b/(L1+L2) b		0.5453		0.5776	0.5569
L2b/(L1+L2) b		0.4547		0.4224	0.4431

CHAPTER IV
DISCUSSION AND CONCLUSION

Starspot Model

The starspot idea was first advanced in the modern era by Kron (1947, 1952) as an explanation for the photometric irregularities of AR Lac and YY Gem. However, quantitative details were lacking in Kron's discussions of systems.

Later Krzeminski (1969) analyzed the hypothesis that cause of the variation in BY Dra was the presence of starspots on the surface with a temperature of a few hundred degrees cooler than the surrounding photosphere.

By analogy with sunspots, these starspots are thought to be the sites of locally strong magnetic fields. Arguments based on flare energy density and on starspot structure suggest that the surface field strengths in starspots may be large, on the order of 10-40 kilogauss (Mullan 1974). Properties of starspots on the surface of a low mass star have been inferred from the light curves of BY Dra and BS CVn type binaries.

Details of the starspot modeling procedure have been given by several authors. Let us review methods for dealing with light curves which may exhibit the effects of starspots.

Torres and Ferraz Mello (1973) fitted the V and R observations of AU Mic using a two-spot model. In their spot model, the ratio (G) between the effective projected area of the spot and the effective area of the star's disk was calculated from the amplitude (A) of the brightness variations:

$$A(\lambda) = -2.5 \log \left(1 - \left(1 - \frac{B(T, \text{spot})}{B(T, \text{star})}\right) * G(\lambda)\right)$$

To reproduce the shape of the asymmetric light curve, they assumed the spotted area was splitted into two separated regions.

Bopp and Evans (1973) discussed the photometric behavior of the flare stars BY Dra and CC Eri due to the rotation of a star having a cool, dark spot on its surface. A model for the spotted area was developed, incorporating four free parameters: longitude extent, latitude extent, limb darkening, and temperature. First they established that spots with considerable extent in latitude could reproduce the general shape of the light curves, then produced several dozen synthetic light curves, varing all three parameters: the limb darkening coefficient, spot area and spot temperature. They quickly realized that the observations could be reproduced with only a narrow range of values. In all the synthetic light curves, it was assumed that the

variations in V were due to a single spot on the primary component.

Friedemann and Gurtler (1975) presented a sample of typical light curves illustrating the effect of variation in polar distance and temperature of the spot, and the inclination of the system.

Budding (1977) developed the surface integrals describing the photometric contributions of the spot in an analytic form, with the simplifying assumption of a single spot with circular outline. In his work, only the geometric parameters (latitude of spot center, inclination of the rotational axis and angular extent of the spot) were optimized to get a curve fit.

Eaton and Hall (1979) have used a starspot model to interpret the light curves of three RS CVn type systems. In their model, the effect of the spots on the light curve was enhanced by putting in a lot of small spots rather than a few larger ones, in the assumed ranges of the latitude of both above and below the equator. The light curve fitting was done by adjusting the geometrical and photometrical parameters. They adopted a subjective parameter adjustment. No numerical procedure was used to decide what parameter changes to make.

Vogt (1981) presented a method for deriving both starspot temperature and effective area from the range of standardized V and R light curves obtained through the

rotation period. This method determined the V-R color difference between a star and a spot and a spot temperature was then derived through the (V-R, T) calibration of Johnson (1966) and Hoxie (1973). For the spot area, the method actually derives a parameterized effective area which represents the fractional area of the disk covered by the spot. Computed light curves for all possible combinations of spot size, location and stellar inclination were produced. To find regions of "best fit", a goodness of fit index was computed for each curve and the fit indices were then plotted as a three-dimensional surface as a function of spot radius and spot latitude.

Bopp and Noah (1980) reproduced the asymmetric light curves of non-eclipsing RS CVn binaries Lambda And and II Peg, using the method outlined by Bopp and Evans (1973). Dorren et al. (1981) fitted their light curves of V711 Tau using the starspot model of Torres and Ferraz Mello (1973).

Reviewing the previous starspot models, we find that these models may share a couple of weak points. First are proximity effects. All models did not include the proximity effects, which can not be neglected in a binary system. Second is how to find the spot parameters uniquely. Their spot model can reproduce light curves from given parameters. However, the spot parameters cannot be found from the observations directly. Therefore, some of the investigators produced several dozens of light curves for possible

combinations of the spot parameters and chose the best fit, and others adopted sujective parameter corrections. In the case of the former, it is difficult tc define a possible combination. The latter may lead to a wrong parameter combination even if a fit is perfect. Despite these handicaps, the success of the spot model has been impressive.

In this dissertation we paid attention to the inverse problems: to find the spot parameters from the observations, considering the proximity effects. The WD program, adopted in this dissertation, includes all possible proximity effects: rotational and tidal distortion, the reflection effect, limb darkening effect, and grvity darkening effect. The spot parameters are determined from the observations by the method of differential correction. Details of this procedure have already been shown in chapter 3.

Properties of Starspots

RS Canum Venaticorm

Summaries of the parameter evolutions for each epoch are tabulated in Tables 30 through 37 respectively. The luminosities (L1, L2) in Tables 30 through 37 are 4 pi steradian luminosities and are about 4 pi times as large as the computed light values in IC program, which are intensities per steradian. The fits for each epoch are shown in Figures 6 through 12 respectively. The solid represents a

computed light curve and cross marks are the observations in each figure.

The 1949 Spots. The observations for this epoch form the first light curve covering the whole phase and show a sinusoidal variation. The 13th iteration gives the final values of the parameters. The spot parameters, the temperature of the primary star, and relative luminosities, are adjusted for this epoch. The latitudes of the 1949 spots are somewhat high compared to other epochs: 53 degrees and 43 degrees, respectively. Eaton and Hall (1979) attempted to fit the starspot model to this epoch. Their model describes very small spots distributed between the limiting latitudes of 40 degrees and especially concentrated at the longitude of 35 and 120 degrees in the coordinate system of WD program. These values are little different from our spot parameters. However, this result cannot be compared directly with our result because their model is different from our model.

The 1963 Spots. Thirteen iterations were done for this epoch. The fit around the secondary eclipse is not perfect due to the lack of observations. The first spot is much larger and cooler than the first spot of the 1949 epoch, because of the large amplitude of the sinusoidal wave for the 1963 epoch. The two spots are approximately 100 degrees apart in longitude.

The 1964 Spots. The fit of this epoch is almost perfect even though fewer iterations (11) were done than for the other epochs. The spot distribution of this epoch caused the displacement of the secondary minimum. This confirms Catalano and Rodono's suggestion that such a displacement is not attributed to orbital eccentricity. Each spot in this epoch migrated in longitude in a direction decreasing relative to the position of the 1963 spots. The two spots are 74 degrees apart in longitude. Eaton and Hall (1979) reproduced the light curve for this epoch with the same starspot model as the 1949 epoch. In their model, spots are distributed between the latitude of 40 degrees and especially concentrated at the longitude of 302 degrees.

The 1965 Spots. Each spot is at high latitude: 43 degrees and 41 degrees. The fit is almost perfect. The shape of the light curve is different from those of the 1963 epoch and the 1964 epoch, but each spot continued to migrate in the same direction. The separation of the two spots in longitude becomes 100 degrees again.

The 1966 Spots. The spot distribution of this epoch distorted the shape of the primary eclipse. Catalano and Rodono (1974) indicated that it was difficult to determine good times of minima due to the asymmetry of the photometric eclipse. They determined the times of the minimum light as 2439215.562 for the 1966 epoch. The time of the minimum

light can be inferred from the phase shift parameter in the DC program which gives the phase of conjunction. If the spots had not disturbed the primary eclipse, the phase of conjunction would have been the time of the minimum light. The phase shift of 0.012 was used to computed a new time of the minimum light for this epoch. This time of minimum light, 2439215.5665, is 0.0045 day later than that of Catalano and Rodono. Each spot continued to migrate in the same direction(retrograde).

The 1975 Spots. Multi-color observations were used to find the spot parameters for this epoch. Two spots are located at the same latitude and 100 degrees apart in longitude. Even if the 1975 spot distribution reproduces the shape of the sinusoidal wave, the fit of the secondary eclipse is not perfect. It was very difficult to improve the fit of the secondary eclipse with the simple two circular spots. Either more spots or a different shape of spot may be needed for a better fit of the secondary eclipse. However the fit is good enough to find the location of the big spot group.

The 1976 Spots. Multi-color observations were also used for this epoch. The sizes of the 1976 spots are the same as those of the 1975 spots, while temperatures of the 1976 spots are 500-700 degrees cooler than the temperatures of the 1975 spots, which caused a larger amplitude than in

1975. The two spots are 87 degrees apart in lingitude. Each spot still migrates in longitude, in a decreasing direction.

The 1979 Spots. The three color observations were used for this epoch. Caton (1981) reproduced the 1979 light curves using the WD light curve program. Caton indicated that five spots were required to reproduce the distortion wave. Caton got a good fit for the secondary eclipse. However, he had an inconsistent fit for the primary: at the primary minimum, computed values are higher than the observations for the visual light curve, while computed values are lower than the observations for blue light. Such an inconsitency seems to be due to subjective parameter corrections. Because the DC program permits simultaneous fitting of an arbitrary number of light curves, our two-spot model yields better fits to the three-color observations than Caton's five spot model. Each spot continues to migrate in longitude, in a decreasing direction.

TABLE 30

Summary of Parameter Adjustments for ES CVn (epoch=1949)

Iteration	1	5	7	9	11	13
Co-Lt1	60.00	60.00	60.00	40.00	37.57	37.57
Lg1	70.00	25.78	34.95	38.73	38.73	47.62
R1	25.00	28.40	25.32	32.25	29.08	29.08
TF1	0.700	0.703	0.703	0.703	0.739	0.739
Co-Lt2	60.00	60.00	56.55	56.55	47.02	47.02
Lg2	170.00	173.65	174.63	175.70	175.70	169.77
R2	25.00	43.90	43.90	43.80	43.80	43.70
TF2	0.700	0.812	0.811	0.811	0.898	0.898
PS	0.133	0.133	0.133	0.133	0.133	0.133
T1	6950	6950	6950	7050	7650	7650
T2	4600	4600	4600	4600	4600	4600
Pot1	9.73	9.73	9.73	9.73	9.73	9.73
Pot2	5.23	5.23	5.23	5.23	5.23	5.23
L1v	7.400	7.935	7.705	7.930	8.432	8.441
L2v	Coupled					
D	0.5183	0.0138	0.0102	0.0086	0.0018	0.0016

TABLE 31

Summary of Parameter Adjustments for ES CVn (epoch=1963)

Iteration	1	5	7	10	12	13
Co-Lt1	90.00	90.00	90.00	50.00	56.58	51.91
Lg1	280.00	295.79	292.64	288.99	288.99	288.99
R1	25.00	32.51	32.18	32.18	35.46	38.46
TF1	0.700	0.660	0.564	0.543	0.543	0.543
Co-Lt2	90.00	90.00	90.00	50.00	50.00	50.00
Lg2	342.15	32.64	27.91	27.91	27.91	27.91
R2	31.50	31.50	35.42	42.66	42.66	43.78
TF2	0.780	0.780	0.890	0.850	0.854	0.854
PS	0.000	0.019	0.019	0.019	0.019	0.019
T1	6810	7120	7120	7090	7090	7070
T2	4600	4600	4600	4600	4600	4600
P1	9.73	9.73	9.73	9.73	9.73	9.73
P2	5.23	5.23	5.23	5.23	5.23	5.23
L1v	7.1100	7.5180	7.6950	7.6950	7.6950	7.6950
L2v	Coupled					
D	3.3497	0.1575	0.0428	0.0670	0.0266	0.0253

TABLE 32

Summary of Parameter Adjustments for BS CVn (epoch=1964)

Iteration	1	3	5	7	9	11
Co-Lt1	90.00	90.00	90.00	60.00	77.37	74.20
Lg1	288.30	254.02	265.48	267.43	267.43	267.43
R1	25.00	25.00	30.35	30.35	30.35	30.35
TF1	0.700	0.700	0.692	0.692	0.692	0.692
Co-Lt2	90.00	90.00	90.00	60.00	65.63	82.10
Lg2	342.15	342.15	345.84	345.84	341.80	341.80
R2	31.50	31.50	33.34	33.34	33.34	33.34
TF2	0.780	0.780	0.795	0.816	0.816	0.828
PS	0.000	0.017	0.017	0.017	0.017	0.017
T1	6810	6950	6950	6950	6950	6950
T2	4600	4600	4600	4600	4600	4600
P1	9.73	9.73	9.73	9.73	9.73	9.73
P2	5.23	5.23	5.23	5.23	5.23	5.23
L1v	7.1100	7.3920	7.4600	7.4600	7.4600	7.4600
L2v	Coupled					
D	2.4498	0.0354	0.0082	0.0239	0.0114	0.0067

TABLE 33

Summary of Parameter Adjustments for BS CVn (epoch=1965)

Iteration	1	5	7	9	11	13
Co-Lt1	90.00	90.00	90.00	50.00	46.50	46.50
Lg1	205.43	212.98	206.70	210.42	210.42	210.42
R1	22.37	19.92	21.31	29.36	29.83	30.11
TF1	0.700	0.630	0.558	0.558	0.600	0.600
Co-Lt2	90.00	90.00	90.00	41.93	50.37	48.52
Lg2	306.15	309.97	313.70	313.70	313.70	313.70
R2	32.50	34.53	34.53	55.20	49.36	51.09
TF2	0.780	0.780	0.700	0.848	0.848	0.853
PS	0.014	0.015	0.015	0.015	0.015	0.015
T1	6810	7180	7180	6990	6990	6990
T2	4600	4600	4600	4600	4600	4600
Pot1	9.73	9.73	9.73	9.73	9.73	9.73
Pot2	5.23	5.23	5.23	5.23	5.23	5.23
L1v	6.9800	7.5910	7.3760	7.4030	7.4140	7.4370
L2v	Coupled					
D	0.2927	0.0309	0.0305	0.0217	0.0110	0.0104

TABLE 34

Summary of Parameter Adjustments for RS CVn (epoch=1966)

Iteration	1	5	8	10	12	16
Co-Lt1	90.00	90.00	90.00	34.83	30.16	30.16
Lg1	144.30	191.69	199.58	199.58	199.58	200.55
R1	25.00	25.00	25.82	44.33	45.37	47.57
TF1	0.700	0.700	0.586	0.586	0.586	0.728
Co-Lt2	90.00	90.00	90.00	50.00	66.57	58.18
Lg2	324.15	284.15	278.58	278.58	275.13	275.13
R2	31.50	31.50	30.65	30.65	30.65	33.82
TF2	0.780	0.780	0.880	0.859	0.897	0.891
PS	0.000	0.012	0.012	0.012	0.012	0.012
T1	6810	6810	6810	6810	6810	6810
T2	4600	4600	4600	4600	4600	4600
Pot2	5.23	5.23	5.23	5.23	5.23	5.23
L1	7.1100	7.1080	7.1080	7.1080	7.1040	7.0980
L2	Coupled					
D	3.6318	0.0711	0.0387	0.0578	0.0238	0.0209

TABLE 35

Summary of Parameter Adjustments for RS CVn (epoch=1975)

Iteration	1	5	7	9	11	13
Co-Lt1	90.00	90.00	60.00	71.36	71.36	71.36
Lg1	126.00	163.30	163.30	163.30	167.10	167.10
R1	25.00	25.00	25.00	25.00	25.00	25.18
TF1	0.780	0.760	0.810	0.810	0.810	0.810
Co-Lt2	90.00	90.00	60.00	70.92	70.92	70.92
Lg2	234.00	255.35	260.15	260.15	261.07	261.87
R2	38.00	31.50	31.50	31.50	31.50	31.50
TF2	0.820	0.820	0.780	0.780	0.780	0.780
PS	0.000	0.000	0.000	0.000	0.000	0.000
T1	6700	6700	6810	6810	6810	6810
T2	4600	4600	4600	4600	4600	4600
Pot1	9.73	9.73	9.73	9.73	9.73	9.73
Pot2	5.23	5.23	5.23	5.23	5.23	5.23
L1v	6.9970	6.9250	7.1100	7.1000	7.0380	7.0929
L2v	Coulped					
D	0.0506	0.0159	0.02169	0.01627	0.01324	0.01291

TABLE 36

Summary of Parameter Adjustments for RS CVn (epoch=1976)

Iterartion	1	5	8	11	14	16
Co-Lt1	90.00	90.00	90.00	56.32	62.73	62.73
Lg1	163.30	120.35	118.39	118.39	144.69	144.69
R1	25.00	24.88	24.88	28.16	28.77	28.77
TF1	0.810	0.810	0.750	0.750	0.749	0.749
Co-Lt2	90.00	90.00	90.00	66.12	76.56	76.56
Lg2	260.15	233.10	230.23	230.23	231.58	231.58
R2	31.50	31.50	32.54	32.54	32.19	32.19
TF2	0.780	0.690	0.690	0.697	0.657	0.657
PS	0.000	0.0000	-0.002	-0.002	-0.002	-0.002
T1	6810	6840	6840	6840	6840	6840
T2	4600	4600	4600	4600	4600	4600
Pot1	9.73	9.73	9.73	9.73	9.73	9.73
Pot2	5.23	5.23	5.23	5.23	5.23	5.23
L1v	7.1100	7.0910	7.1410	7.1630	7.1570	7.1526
L2v	Coupled					
L1b	8.6290	8.2540	8.3010	8.3430	8.3240	8.3141
L2b	Coupled					
D	0.3250	0.4034	0.0296	0.03605	0.0248	0.0213

TABLE 37

Summary of Parameter Adjustments for RS CVn (epoch=1979)

Iterartion	1	5	7	9	11	12
Co-Lt1	90.00	60.00	60.00	60.00	51.43	51.43
Lg1	288.99	306.97	325.21	318.88	318.88	318.88
R1	30.47	22.91	26.31	25.67	30.53	30.53
TF1	0.540	0.743	0.653	0.653	0.720	0.691
Co-Lt2	90.00	60.00	60.00	62.85	62.85	62.85
Lg2	27.91	33.60	66.88	69.37	69.37	69.37
R2	42.66	25.66	22.37	22.37	23.12	23.12
TF2	0.916	0.663	0.663	0.663	0.663	0.663
PS	0.019	-0.001	-0.001	-0.001	-0.001	-0.001
T1	7090	6790	6790	6800	6800	6840
T2	4560	4560	4560	4506	4560	4370
Pot1	9.73	9.73	9.73	9.73	9.73	9.73
Pot2	5.23	5.23	5.23	5.23	5.23	5.23
L1v	7.4570	7.6570	7.5330	7.5670	7.6100	7.6400
L2v	Coupled					
L1b	8.6520	8.9580	8.8040	8.8260	8.8700	8.9150
L2b	Coupled					
L1u	9.0020	10.0520	9.9340	9.9440	9.9840	10.0430
L2u	Coupled					
D	1.8257	0.0278	0.0129	0.0116	0.0111	0.0110

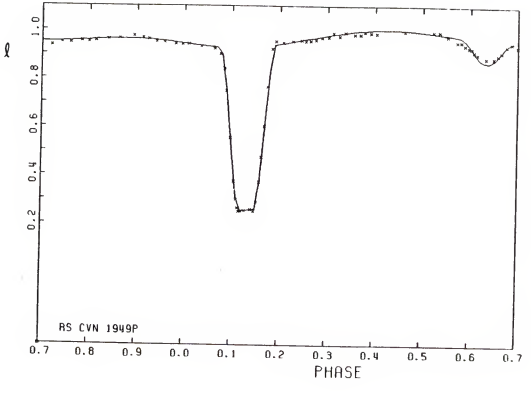

Figure 6: The Fit to the Light Curve of RS CVn, 1949 Epoch.

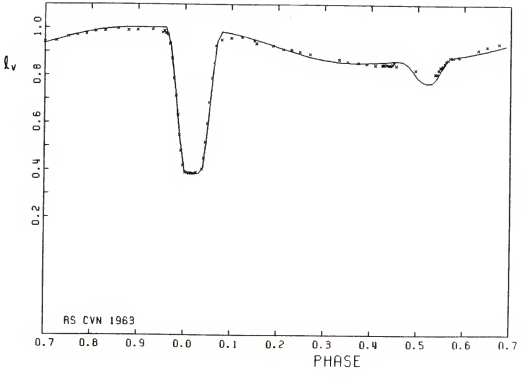

Figure 7: The Fit to the Light Curve of RS CVn, 1963 Epoch.

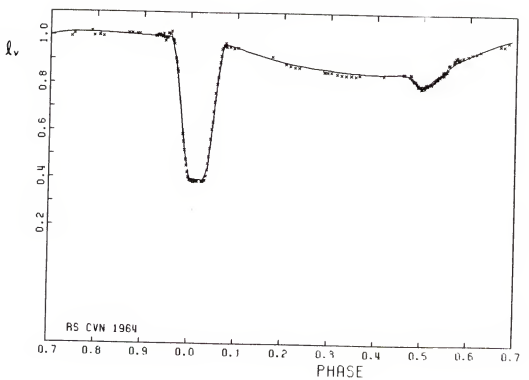

Figure 8: The Fit to the Light Curve of RS CVn, 1964 Epoch.

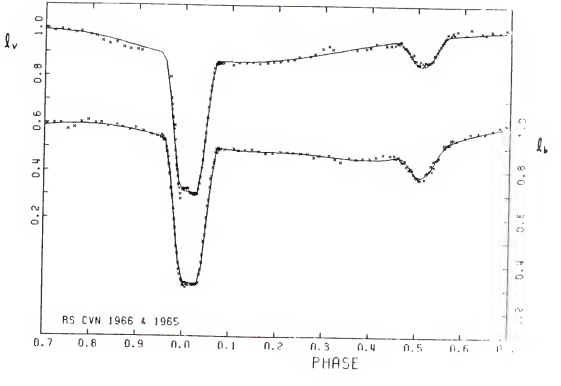

Figure 9: The Fit to the Light Curves of RS CVn, 1965 to 1966 Epoch.

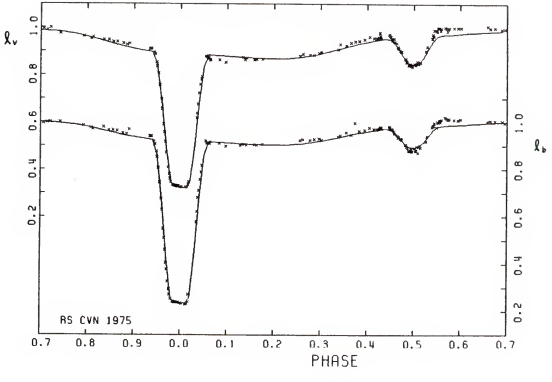

Figure 10: The Fit to the Light Curves of RS CVn, 1975 Epoch.

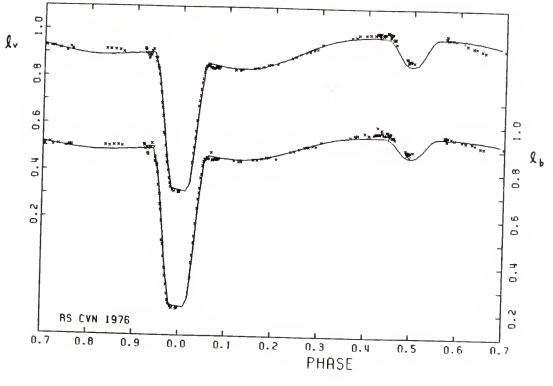

Figure 11: The Fit to the Light Curves of RS CVn, 1976 Epoch.

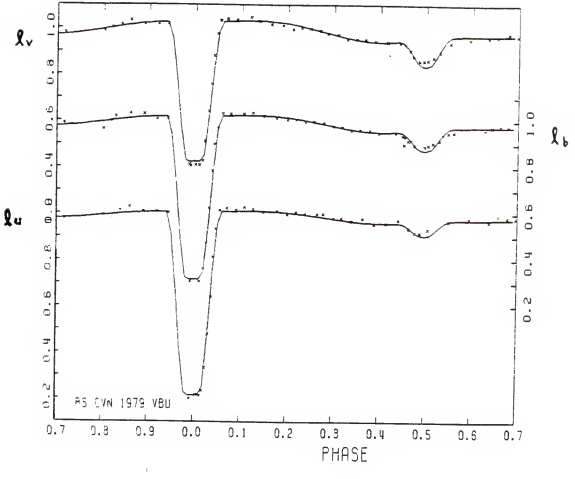

Figure 12: The Fit to the Light Curves of BS CVn, 1979 Epoch.

AR Lacertae

No starspot model has previously been attempted for AR Lac. A distortion wave for AR Lac was not seen until the 1978 epoch. However, it is very clear that the light level before the primary minimum is different from that before the secondary minimum for all epochs before 1978. Also such a wave could not be found before 1978 because of a lack of observations outside eclipse. The summaries of the parameters evolutions for each epoch are tabulated in Tables 38 through 43, respectively. The luminosities (L1, L2) in Tables 38 through 43 are 4 pi sterradian luminosities and are about 4 pi times as large as the computed light values in LC program, which are intensities per sterradian. The fits for each epoch are shown in Figures 13 through 18, respectively. A solid line represents a computed light curve and cross marks are the observations in each figure.

The 1972 Spot. Only one spot model was applied for this epoch, because the phase coverage of this epoch can not give much information about the spot properties. As can be seen in Figure 13 the light level at the phase 0.34-0.41 is higher than that at phase 0.10 or 0.90. Thus the initial longitude of the spot was adopted as 180 degrees: the spot faces toward the observer at the primary minimum. The latitude of a spot was fixed at 20 degrees south (co-latiitude is 70 degrees). The one-spot model was good enough to reproduce the light curves of this epoch.

The 1975 Spots. The observations in this epoch have a large scatter compared to other epochs. A sinusoidal distortion wave is not shown in the light curve; however the light level at phase 0.75 is higher than that at phase 0.25. The two spots are 140 degrees apart in longitude and the two-spot model reproduces the light curve much better than the one-spot model.

The 1977 Spots. It is hard to tell whether the light curve of this epoch shows distortion wave, because it lacks observations between the phase, 0.12 and 0.32, and the phase, 0.57 and 0.78. However, the light curve also shows a level difference between maxima. The two spots are 100 degrees apart in longitude and on the same latitude (fixed).

The 1978 Spots. The light curve of this epoch is the first one to cover the whole phase for AR Lac, and to show a sinusoidal distortion wave, similar to that seen in the light curves of RS CVn. The amplitude of the distortion wave was measured as 0.09 magnitude by Kurutac et al. (1981). Our two-spot model was successful in reproducing an asymmetric distortion wave as indicated by Kurutac et al. (1981). The spot latitudes for this epoch were adjusted, because of good phase coverage of the light curves in multi-color. The two spots are 74 degrees apart in longitude.

The 1980 spots. A single color observation was used to find spot parameters for this epoch. The phase coverage of

this light curve is better than that of the 1978 epoch except for the secondary eclipse. The light curve shows a sinusoidal wave; however, the shape of the light curve is different from that of the 1978 epoch. Ertan et al. (1982) indicated that the wave-like distortion has two maxima, the first at phase 0.7 and the second at phase 0.25 and, therefore two minima. The two-spot model gives a better result than a one-spot model. Two spots are on the same latitude (fixed) and approximately 120 degrees apart in longitude. This spot distribution reproduces the sinusoidal distortion wave and the fit for this epoch is almost perfect.

The 1981 Spots. The light curves for this epoch show a sinusoidal wave outside eclipse, and cover the most phases in multi colors. The two-spot model gives a little better result than the one-spot model. The two spots are separated 60 degrees in longitude. The latitudes of the spots were also adjusted for this epoch. It is interesting that one of the spots is much smaller and warmer for this epoch, while the two spots are nearly the same size for all other epochs.

TABLE 38

Summary of Parameter Adjustments for AR Lac (epoch=1972)

Iteration	1	2	3	4	5
Co-Lt1	70.00	70.00	70.00	70.00	70.00
Lg1	180.00	180.00	180.00	168.07	168.07
R1	15.00	11.18	11.18	11.18	10.88
TF1	0.850	0.850	0.850	0.880	0.880
PS	0.001	0.001	0.001	0.001	0.001
T1	5400	5400	5400	5400	5400
T2	4500	4517	4567	4567	4567
Pot1	7.0713	7.0713	7.0713	7.0713	7.0713
Pot2	4.2720	4.2720	4.2720	4.2720	4.3056
L1v	4.8900	4.8200	4.6768	4.6717	4.7355
L2v	Coupled				
L1b	5.7120	5.6184	5.4202	5.4150	5.4828
L1b	Coupled				
D	0.0296	0.0244	0.0081	0.0072	0.0064

TABLE 39

Summary of Parameter Adjustments for AR Lac (epoch=1975)

Iteration	1	3	4	5	6	7
Co-Lt1	70.00	70.00	70.00	70.00	70.00	70.00
Lg1	126.00	105.48	74.89	74.89	65.01	65.01
R1	13.00	10.84	10.84	16.90	15.96	15.96
TF1	0.750	0.750	0.750	0.750	0.750	0.691
Co-Lt2	70.00	70.00	70.00	70.00	70.00	70.00
Lg2	284.00	271.25	282.73	284.17	282.71	284.00
R2	15.00	15.00	15.00	15.00	15.00	15.00
TF2	0.800	0.800	0.800	0.800	0.732	0.732
PS	0.999	0.999	0.999	0.999	0.999	0.999
T1	5600	5610	5610	5670	5660	5680
T2	4600	4760	4800	4800	4800	4800
Pot1	7.1013	7.1013	7.1013	7.1013	7.1013	7.1013
Pot2	4.2670	4.3730	4.3730	4.3730	4.3730	4.3730
L1v	4.5830	4.5290	4.5290	4.6960	4.7530	4.7570
L2v	Coupled					
L1b	5.0720	4.9900	4.9900	5.2020	5.2710	5.2740
L2b	Coupled					
D	0.4540	0.1090	0.0375	0.0308	0.0430	0.0290

TABLE 40

Summary of Parameter Adjustments for AR Lac (epoch=1977)

Iteration	1	3	5	6	7	8
Co-Lt1	70.00	70.00	70.00	70.00	70.00	70.00
Lg1	270.00	286.22	297.74	297.74	297.74	297.74
R1	20.00	14.57	14.57	14.57	14.782	14.782
TF1	0.850	0.664	0.664	0.664	0.664	0.664
Co-Lt2	70.00	70.00	70.00	70.00	70.00	70.00
Lg2	90.00	32.33	35.64	35.64	35.64	35.64
R2	20.00	18.90	18.28	18.28	18.28	18.28
TF2	0.800	0.800	0.800	0.800	0.800	0.800
PS	0.999	0.999	0.999	0.999	0.999	0.999
T1	5540	5560	5560	5560	5560	5560
T2	4600	4600	4600	4600	4586	4594
Pot1	7.0713	7.0713	7.0713	7.0713	7.0713	7.0713
Pot2	4.2920	4.2920	4.2920	4.2970	4.2970	4.2970
L1v	5.0260	5.1100	5.1100	5.1220	5.1920	5.1539
L2v	Coupled					
L1b	5.5960	5.6820	5.6820	5.6930	5.7803	5.7288
L2b	Coupled					
D	0.0479	0.0205	0.0198	0.0197	0.0189	0.0184

TABLE 41

Summary of Parameter Adjustments for AR Lac (epoch=1978)

Iteration	1	5	10	12	14	15
Co-Lt1	70.00	54.81	54.81	54.81	66.09	66.09
Lg1	310.00	352.61	13.58	13.58	13.58	355.23
R1	21.00	21.72	16.97	16.97	13.65	13.65
TF1	0.830	0.830	0.809	0.761	0.761	0.709
Co-Lt2	70.00	70.00	70.00	70.00	70.00	70.00
Lg2	250.00	271.46	288.05	281.14	281.14	281.14
R2	20.00	20.00	20.44	20.44	20.31	18.41
TF2	0.800	0.890	0.832	0.832	0.832	0.832
PS	0.990	0.998	0.998	0.998	0.998	0.998
T1	5600	5430	5551	5551	5551	5551
T2	4550	4550	4630	4630	4630	4630
Pot1	7.0713	7.0713	7.0713	7.0713	7.0713	7.0713
Pot2	4.3720	4.3020	4.3470	4.3470	4.3118	4.3144
L1v	5.0000	4.8010	4.9489	4.9085	4.8345	4.8344
L2v	Coupled					
L1b	5.6000	5.5710	5.6346	5.5669	5.4885	5.4874
L2b	Coupled					
D	0.5590	0.0680	0.0450	0.0282	0.0210	0.0189

TABLE 42

Summary of Parameter Adjustments for AR Lac (epoch=1980)

Iteration	1	5	13	19	21	23
Co-Lt1	80.00	60.00	70.00	70.00	70.00	70.00
Lg1	180.39	158.89	139.50	158.89	149.93	142.32
R1	20.00	20.00	18.43	20.00	17.08	15.94
TF1	0.800	0.800	0.837	0.850	0.850	0.846
Co-Lt2	90.00	60.00	70.00	70.00	70.00	70.00
Lg2	322.00	360.00	223.72	310.14	268.97	268.97
R2	20.00	30.00	15.00	15.00	17.02	17.02
TF2	0.800	0.700	0.930	0.900	0.900	0.925
PS	0.990	0.995	0.995	0.995	0.995	0.995
T1	5350	5350	5430	5520	5520	5569
T2	4550	4550	4550	4550	4550	4550
Pot1	7.0713	7.0713	7.0713	7.0713	7.0713	7.0713
Pot2	4.3720	4.3720	4.2970	4.2646	4.2646	4.2882
L1v	5.1000	4.7920	4.8770	5.0340	5.0650	5.1840
L2v	Coupled					
D	0.1866	0.0239	0.0120	0.0179	0.0085	0.0070

TABLE 43

Summary of Parameter Adjustments for AR Lac (epoch=1981)

Iteration	1	3	8	10	12	14
Co-Lt1	80.00	80.00	67.41	67.41	67.41	67.41
Lg1	180.00	196.56	248.51	248.51	247.35	247.35
B1	20.00	14.00	17.61	17.61	18.91	18.91
TF1	0.800	0.800	0.792	0.792	0.792	0.792
Co-Lt2	90.00	90.00	70.00	70.00	70.00	70.00
Lg2	322.13	297.02	145.00	145.00	173.05	186.86
R2	20.00	20.00	15.00	9.70	9.70	9.70
TF2	0.850	0.850	0.880	0.880	0.955	0.933
PS	0.990	0.993	0.993	0.993	0.993	0.993
T1	5350	5350	5350	5350	5350	5368
T2	4550	4550	4550	4550	4550	4550
Pot1	7.0713	7.0713	7.0713	7.0713	7.0713	7.0713
Pot2	4.3720	4.3720	4.3720	4.3793	4.3793	4.3532
L1v	4.8000	4.6090	4.6010	4.6041	4.6192	4.6015
L2v	Coupled					
L1b	5.4000	5.2890	5.2936	5.2737	5.2921	5.2833
L2b	Coupled					
D	0.1724	0.0358	0.0338	0.0245	0.0232	0.0218

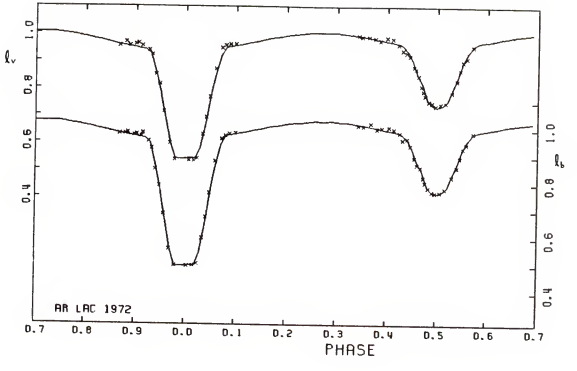

Figure 13: The Fit to the Light Curves of AR Lac, 1972 Epoch.

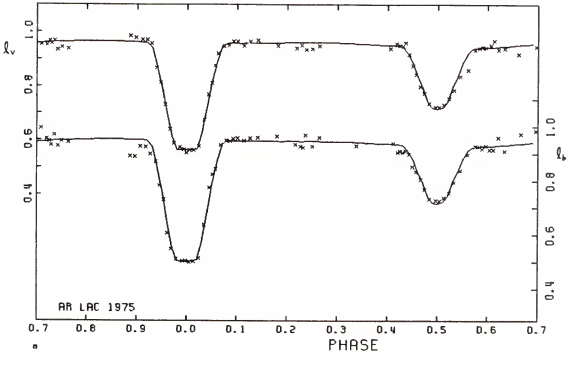

Figure 14: The Fit to the Light Curves of AR Lac, 1975 Epoch.

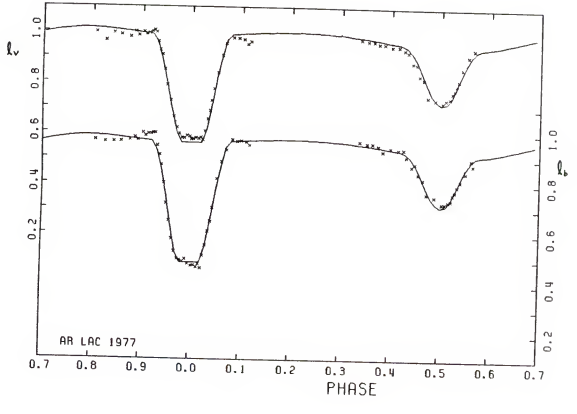

Figure 15: The Fit to the Light Curves of AR Lac, 1977 Epoch.

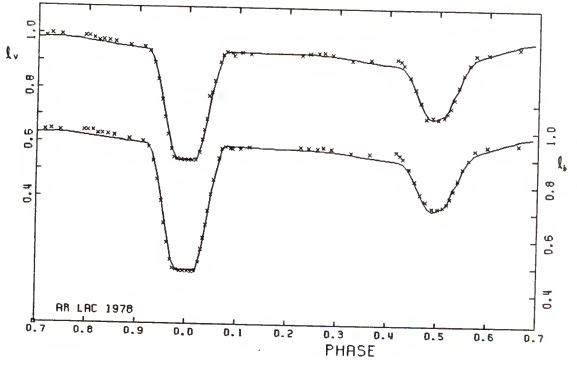

Figure 16: The Fit to the Light Curves of AR Lac, 1978 Epoch.

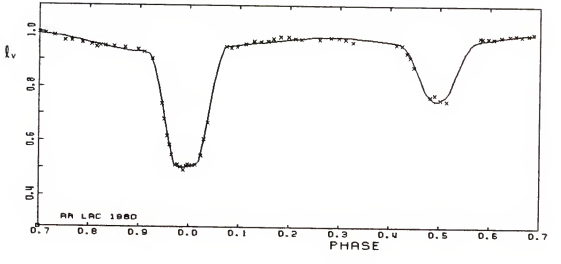

Figure 17: The Fit to the Light Curve of AR Lac, 1980 Epoch.

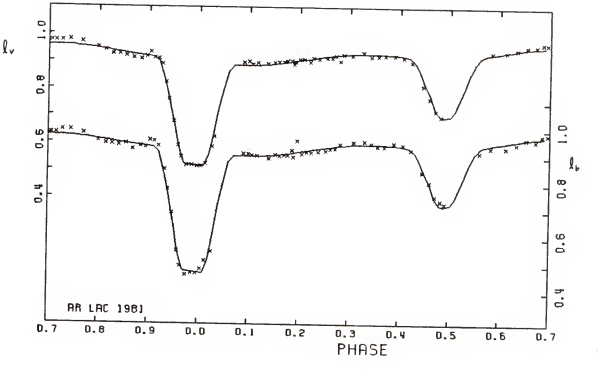

Figure 18: The Fit to the Light Curves of AR Lac, 1981 Epoch.

V711 Tauri

Because V711 Tau is a non eclipsing binary, the spot latitude was not adjusted for all epochs. Summaries of parameter adjustments of V711 Tau for all epochs are tabulated in Tables 44 through 52. The luminosities (L1, L2) in Tables 42 through 52 are 4 pi times as large as the computed light values in IC program, which are intensities per sterradian. The fit of the two spot model for each epoch is plotted in Figures 19 through 27.

The 1963 Spots. The light variation of V711 Tau was discovered by the observations of this epoch. Only eight observation show a quasi-sinuscidal wave. The two-spot model could reproduce the shape of the distortion wave for this epoch. The two spots are separated 140 degrees in longitude.

The 1975 Spots. The two-spot model gives a perfect fit of an asymmetric distortion wave for this epoch. The two spots are approximately 90 degrees apart in longitude.

The 1976 Spots. The two spots had migrated 45 degrees and 10 degrees from the positions of the spots of the 1975 epoch, in such manner that the longitude decreases. This confirms the retrograde migration of the distortion wave found by Landis et al. (1978).

The 1977 Spots. The sum of the squares of weighted residuals does not give significant difference between the two-spot model and the one-spot model. However the fit of the two-spot model is significantly better than the fit of one-spot model graphically. One large spot migrated 15 degrees in longitude from the position of the corresponding spot of the 1976 epoch, in such manner that the longitude increased, while a small spot migrated 30 degrees in longitude the other way so that the longitude decreased. The former is inconsistent with the retrograde migration. Dorren et al. (1981) fitted their light curves (wavelengths are 6563A and 7790A) using the starspot model of Torres and Ferraz Mallo. Their two spots are located at 239 degrees and 7 degrees in our longitude system (they used a different definition of longitude), while our two spots are at 306 degrees and 59 degrees in longitude. There are approximately 60 degrees difference in the spot longitude between two models. Such a difference is because the shape of the light curve adopted in this dissertation is different from that of Dorren et al.'s light curve.

The 1978B Spot. The one-spot model was used for this epoch, because the one-spot model adquately reproduces light curve of this epoch. The observation of this epoch were made during the large radio outburst (Feldman et al., 1978) of February 1978. A spot migrated in such a manner that the longitude increased. This confirms a direct migration of the wave found by Chambliss et al. (1978).

The 1978E Spots. Multi-color observations were used to analyze light curves for this epoch. The amplitude of the wave is much larger than those of the 1978E, 1977 epochs. Thus sizes of spots became larger than those of the previous epochs. The two-spot model gives a perfect fit. Both spots continued to show a direct migration.

The 1979B Spots. The wave did not change its shape or large amplitude significantly compared to the 1978E epoch. The two-spot model gives a better fit than the one spot model. The two spots are approximately 80 degrees apart in longitude. Both spots continued to migrate in such a manner that the longitude increased (direct motion).

The 1979E Spots. Multi-color observations were used to analyze the light curves for this epoch. The two-spot model gives a perfect fit for this epoch. Two spots are located at 129 degrees and 267 degrees of longitude. These spot locations are in good agreement with those of Dorren et al. (1981), who fitted their 6585A wavelength observations using the starspot model of Torres and Herraz Mallo.

The 1981 Spots. Multi-color observations were used for this epoch. However there are not enough observations to analyze the light curvres. The two-spot model gives a significantly better fit than the one-spot model. The two spots are 90 degrees apart in longitude and continued to migrate in such a manner that the longitude decreases.

TABLE 44

Summary of Parameter Adjustments for V711 Tau (epoch=1963)

Iteration	1	3	5	7	9
Lt1	55.00	55.00	55.00	55.00	55.00
Lg1	50.00	48.87	48.87	66.55	68.00
R1	30.00	35.79	31.38	31.38	37.13
TF1	0.750	0.746	0.710	0.710	0.755
Lt2	55.00	55.00	55.00	55.00	55.00
Lg2	90.00	288.00	288.00	288.00	288.00
R2	10.00	10.00	25.26	27.75	27.75
TF2	0.850	0.850	0.850	0.850	0.815
Pot1	5.100	5.100	5.100	5.100	5.100
Pot2	4.362	4.362	4.362	4.362	4.362
L1	6.3860	6.2776	6.3765	6.4741	6.6680
L2	Coupled				
D	0.00071	0.00031	0.00029	0.00015	0.00006

TABLE 45

Summary of Parameter Adjustments for V711 Tau (epoch=1975)

Iteration	1	3	5	7	9
Lt1	55.00	55.00	55.00	55.00	55.00
Lg1	75.00	75.00	75.00	72.03	72.03
R1	20.00	24.35	25.10	25.10	25.10
TF1	0.750	0.750	0.750	0.750	0.745
Lt2	55.00	55.00	55.00	55.00	55.00
Lg2	345.00	345.00	345.00	345.00	345.00
R2	20.00	20.62	20.68	20.68	20.68
TF2	0.750	0.750	0.750	0.750	0.758
Pot1	5.100	5.100	5.100	5.100	5.100
Pot2	4.250	4.250	4.250	4.349	4.349
L1v	5.811	5.811	5.895	6.177	6.160
L2v	Coupled				
D	0.0008	0.0009	0.0004	0.0004	0.0003

TABLE 46

Summary of Parameter Adjustments for V711 Tau (epoch=1976)

Iteration	1	3	5	7	9
Lt1	55.00	55.00	55.00	55.00	55.00
Lg1	55.00	44.94	44.94	44.94	44.94
R1	20.00	20.00	26.20	26.20	26.77
TF1	0.750	0.750	0.750	0.817	0.817
Lt2	55.00	55.00	55.00	55.00	55.00
Lg2	342.00	347.20	328.45	335.14	335.14
R2	20.00	20.00	20.00	21.35	21.35
TF2	0.750	0.750	0.750	0.750	0.742
Pot1	5.100	5.100	5.100	5.100	5.100
Pot2	4.250	4.362	4.362	4.362	4.362
L1v	5.712	6.105	6.165	6.164	6.163
L2v	Coupled				
D	0.0016	0.0014	0.0011	0.0011	0.0010

TABLE 47

Summary of Parameter Adjustments for V711 Tau (epoch=1977)

Iteration	1	3	5	7	9
Lt1	80.00	80.00	80.00	55.00	55.00
Lg1	58.00	41.57	49.52	59.87	59.87
R1	20.00	31.00	31.00	24.50	25.12
TF1	0.750	0.750	0.768	0.768	0.768
Lt2	55.00	55.00	55.00	55.00	55.00
Lg2	300.00	300.00	306.00	306.00	306.00
R2	10.00	10.00	15.00	11.55	11.55
TF2	0.850	0.850	0.850	0.850	0.850
Pot1	5.100	5.100	5.100	5.100	5.100
Pot2	4.250	4.250	4.250	4.250	4.362
L1v	5.550	5.510	5.8118	5.8410	5.9123
L2v	Coupled				
D	0.0040	0.0020	0.0009	0.0005	0.0004

TABLE 48

Summary of Parameter Adjustments for V711 Tau (epoch=1978E)

Iteration	1	3	5	7	9
Lt1	80.00	80.00	80.00	80.00	55.00
Lg1	65.00	67.42	66.70	66.20	66.20
R1	20.00	38.83	33.00	33.00	33.00
TF1	0.750	0.750	0.750	0.851	0.891
Pot1	5.100	5.100	5.100	5.100	5.100
POt2	4.250	4.250	4.250	4.250	4.362
L1v	5.500	5.500	5.480	5.497	5.567
L2v	Coupled				
D	0.0013	0.0017	0.0006	0.0004	0.0003

TABLE 49

Summary of Parameter Adjustments for V711 Tau (epoch=1978E)

Iteration	1	3	5	7	9
Lt1	55.00	55.00	55.00	55.00	55.00
Lg1	108.00	85.15	94.16	94.16	94.16
R1	30.00	30.00	34.05	34.05	34.05
TF1	0.750	0.750	0.750	0.739	0.741
Lt2	55.00	55.00	55.00	55.00	55.00
Lg2	180.00	182.83	180.77	183.65	183.65
R1	30.00	30.00	30.00	29.30	28.87
TF2	0.800	0.750	0.770	0.770	0.770
Pot1	5.100	5.100	5.100	5.100	5.100
Pot2	4.362	4.362	4.362	4.296	4.296
L1v	6.370	6.035	6.451	6.488	6.301
L2v	Coupled				
D	0.0016	0.0006	0.0002	0.0002	0.0001

TABLE 50

Summary of Parameter Adjustments for V711 Tau (epoch=1979B)

Iteration	1	3	5	7	9
Lt1	55.00	55.00	55.00	55.00	55.00
Lg1	100.00	100.00	117.44	117.44	117.44
R1	20.00	35.21	35.21	35.20	34.30
TF1	0.751	0.751	0.751	0.722	0.722
Lt2	55.00	55.00	55.00	55.00	55.00
Lg2	190.00	190.00	195.58	195.58	195.58
R1	20.00	20.00	20.00	20.98	20.98
TF2	0.750	0.750	0.750	0.750	0.805
Pot1	5.100	5.100	5.100	5.100	5.100
Pot2	4.362	4.362	4.362	4.362	4.362
L1v	6.1880	6.3880	6.4740	6.5090	6.5090
L2v	Coupled				
D	0.0016	0.0006	0.0003	0.0002	0.0001

TABLE 51

Summary of Parameter Adjustments for V711 Tau (epoch=1979E)

Iteration	1	3	5	7	9	
Lt1	55.00	55.00	55.00	55.00	55.00	55.00
Lg1	130.00	96.00	139.87	119.77	129.45	129.45
R1	32.00	21.00	21.00	26.83	26.83	26.83
TF1	0.750	0.750	0.600	0.600	0.600	0.683
Lt2	55.00	55.00	55.00	55.00	55.00	55.00
Lg2	210.76	207.96	244.07	233.33	236.83	236.83
R2	30.00	30.00	29.02	29.02	29.02	28.73
TF2	0.750	0.882	0.882	0.882	0.882	0.882
Pot1	5.100	5.100	5.100	5.100	5.100	5.100
Pot2	4.362	4.362	4.362	4.362	4.362	4.362
L1v	6.450	6.174	6.157	6.209	6.227	6.217
L2v	Coupled					
L1b	7.093	6.828	6.785	6.839	6.846	6.839
L1b	Coupled					
D	0.0071	0.0038	0.0023	0.0012	0.0010	0.0009

TABLE 52

Summary of Parameter Adjustments for V711 Tau (epoch=1980)

Iteration	1	3	5	7	9	
Lt1	55.00	55.00	55.00	55.00	55.00	55.00
Lg1	255.00	263.45	263.45	250.91	250.91	250.91
R1	20.00	22.96	22.96	22.96	22.96	23.79
TF1	0.877	0.877	0.879	0.879	0.829	0.829
Lt2	55.00	55.00	55.00	55.00	55.00	55.00
Lg2	347.00	330.48	330.48	340.59	340.59	340.59
R2	20.00	20.00	20.22	20.22	20.22	20.22
TF2	0.850	0.820	0.820	0.820	0.820	0.820
Pot1	5.100	5.100	5.100	5.100	5.100	5.100
Pot2	4.362	4.362	4.362	4.362	4.362	4.362
L1v	5.9260	6.0060	6.0050	6.0100	6.0100	6.0372
L2v	Coupled					
L1b	6.3600	6.4470	6.4460	6.4510	6.4510	6.4783
L2b	Coupled					
D	0.0057	0.0034	0.0034	0.0031	0.0030	0.0029

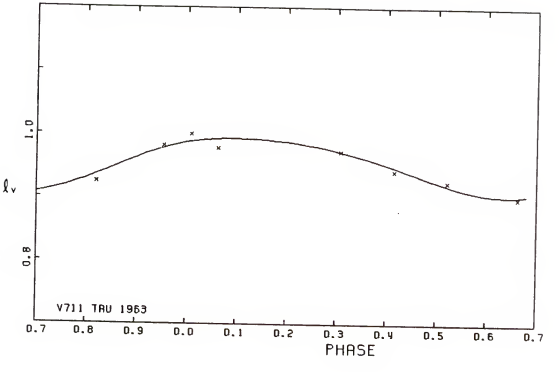

Figure 19: The Fit to the Light Curve of V711 Tau, 1963 Epoch.

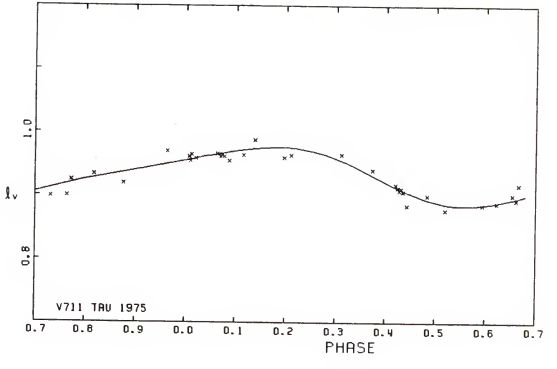

Figure 20: The Fit to the Light Curve of V711 Tau, 1975 Epoch.

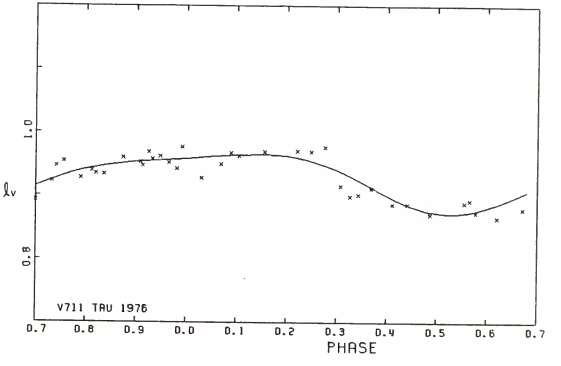

Figure 21: The Fit to the Light Curve of V711 Tau, 1976 Epoch.

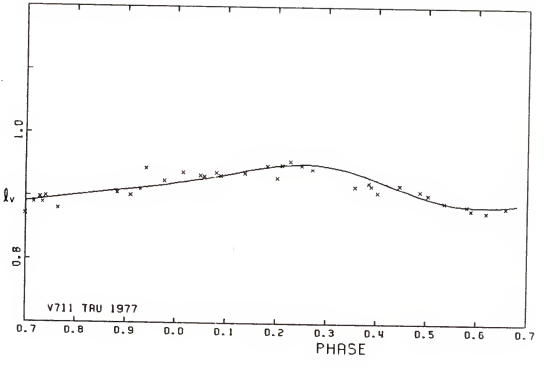

Figure 22: The Fit to the Light Curve of V711 Tau, 1977 Epoch.

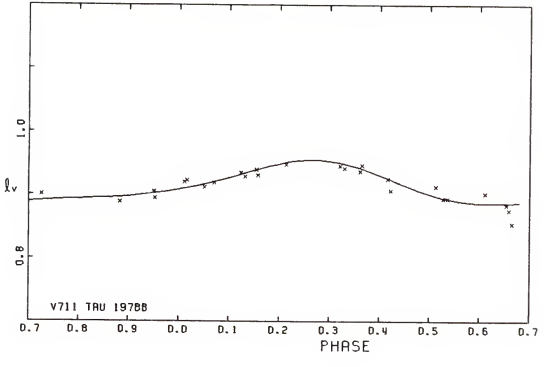

Figure 23: The Fit to the Light Curve of V711 Tau, 1978B Epoch.

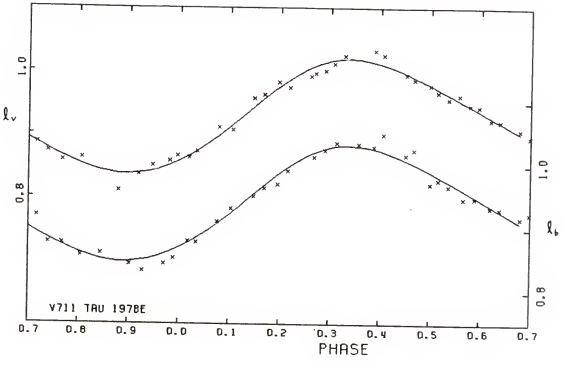

Figure 24: The Fit to the Light Curves of V711 Tau, 1978E Epoch.

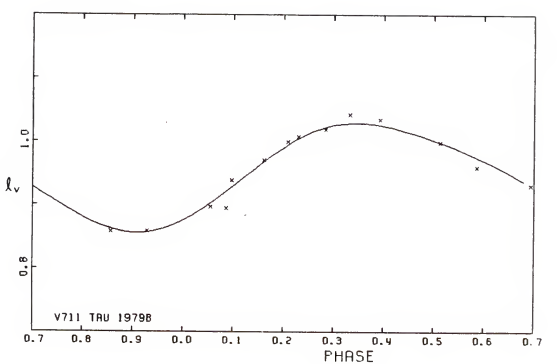

Figure 25: The Fit to the Light Curve of V711 Tau, 1979B Epoch.

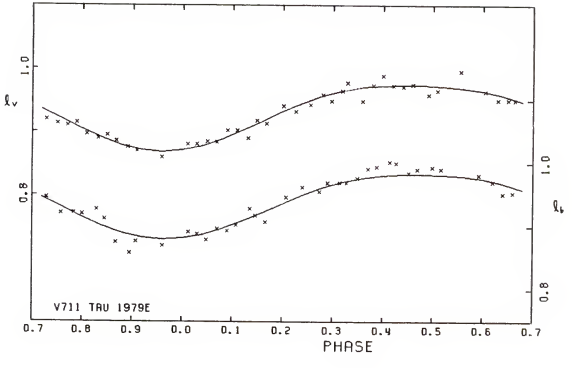

Figure 26: The Fit to the Light Curves of V711 Tau, 1979E Epoch.

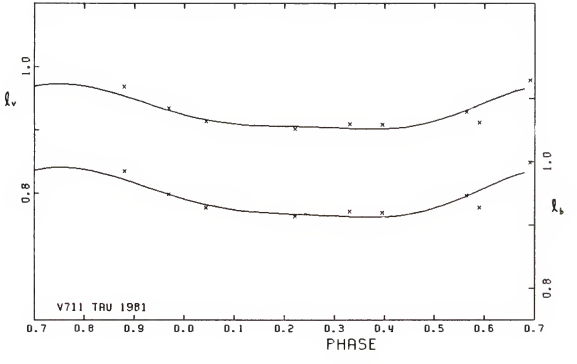

Figure 27: The Fit to the Light Curves of V711 Tau, 1981 Epoch.

Starspot Cycle

RS Canum Vanaticorum

The migration of the distortion wave for RS CVn has been reported by many observers. The period of the migration has been analyzed using the phase of the wave minimum. It is generally accepted that the distortion wave is caused by the starspot. Thus it is interesting to find the starspot cycle using the parameters of the starspots.

As can be seen in Table 20, the latitude of the spot center for RS CVn itself varies from 20 degrees to 60 degrees, while the longitude of the spot center varies all around the star. To investigate the starspot cycle, only the spot longitude variation will be considered. The spot longitude were plotted against time in two cases. For case 1, the 1949 and 1963 spot longitudes are shifted up one cycle, and the 1979 longitudes are shifted down one cycle, in figure 28. For case 2, the 1949 longitudes are shifted up two cycles, those of 1963 are shifted up one cycle and those of 1975, 1976, and 1979 are shifted down one cycle, in Figure 29. A relation of the following form was fitted to the data by the method of least squares:

$$L = w(t-to) + Lo$$
$$w = 2\,pi/P$$

where

w : Angular velocity of a spot (degrees/year)
P : Starspot period in years
t : Time in years
to: Reference time in years
L : Spot longitude in degrees
Lo: Reference spot longitude in degrees

Thus, we have the following result for each spot in each case:

$$L1 = -12.87(t - 1979) + 408.66 \quad \text{(spot 1 in case 1)} \quad \text{--- (2)}$$
S.D. 2.03 30.26

$$L2 = -12.90(t - 1979) + 148.20 \quad \text{(spot 2 in case 1)} \quad \text{--- (3)}$$
S.D. 1.97 29.27

$$L1 = -39.05(t - 1979) + 367.79 \quad \text{(spot 1 in case 2)} \quad \text{--- (4)}$$
S.D. 1.51 22.57

$$L2 = -40.01(t - 1979) + 90.74 \quad \text{(spot 2 in case 2)} \quad \text{--- (5)}$$
S.D. 1.48 22.12

The period of each spot in each case was calculated from the angular velocity of the spot and tabulated in Table 53.

TABLE 53

Period of Each Spot of RS CVn in Each Case.

Spot	Period Case 1	Period Case 2
Spot 1	27.974 yrs	9.219 yrs
Spot 2	27.900 yrs	8.998 yrs
Mean	27.937 yrs	9.109 yrs

As can be seen in both Figure 28 and Figure 29, a 9.1 year cycle in case 2 seems to be preferable to a 27.9 year cycle because the standard errors of angular velocities of spots in case 2 are significantly smaller than those in case 1. This 9.1 year cycle agrees with the period of the

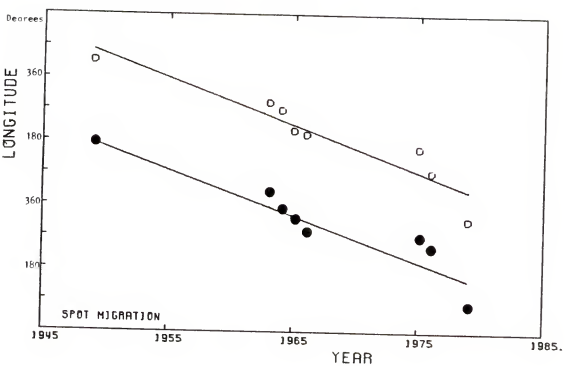

Figure 28: Spot Migration for RS CVn, Case 1.

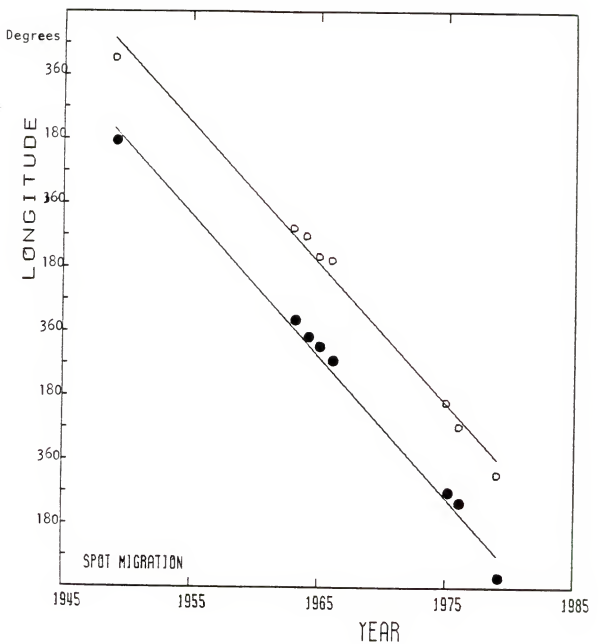

Figure 29: Spot Migration for BS CVn, Case 2.

distortion wave (9.48 years or 722 orbital periods) found by Catalano et al. (1980). Hall (1972) suggested a spot cycle of 23.5 years because he computed 1800 orbital cycles between successive maxima of the distortion wave. Hall's cycle is close to the cycle of 27.9 years in case 1.

AR Lacertae

For AR Lac, the spot longitude variation has also considered to find the starspot cycle. The spot longitudes for the two-spot model were plotted against time in Figure 30. In Figure 30, open circles and filled circles are the spot longitudes of each spot for the two-spot model. The spot of the 1972 epoch was not plotted in Figure 29, because lack of observations outside eclipse does not give much information on the spot parameters and leads us to use one-spot model for the 1972 epoch. Using equation (1), the relations between the spot longitude and time for the two-spot model are determined by the method of a least square analysis:

$$L1 = \underset{\text{S.D. } 3.54}{-31.98}(t - 1981) + \underset{12.45}{248.90} \quad \text{--- (6)}$$

$$L2 = \underset{\text{S.D. } 9.66}{-23.81}(t - 1981) + \underset{34.03}{171.74} \quad \text{--- (7)}$$

The top line was fitted to the open circles and expressed by equation (6). The bottom line was fitted to the filled

circles and expressed by equation (7). The fit does not include the 1972 spot. The starspot cycles for each spot were determined as 11.257 years and 15.180 years from equations (6) and (7), repectively. As can be seen in equations (6) and (7), the standard error for the second spot is much larger than for the first spot. However, these values are similar to the sunspot cycle.

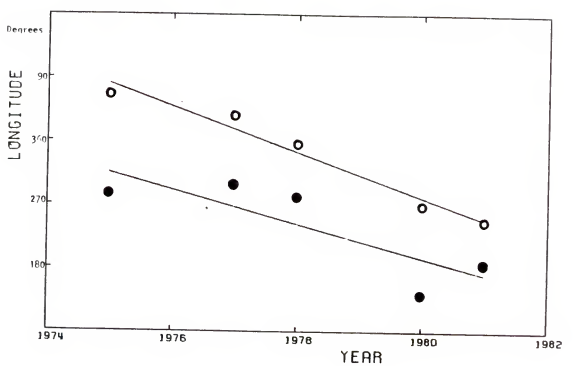

Figure 30: Spot Migration for AR Lacertae.

V711 Tauri

The starspot cycle of V711 Tau has been investigated using the same method as RS CVn and AR Lac. The spot longitudes for the two-spot model and the one-spot model were plotted against time in Figure 31. The open circles and filled circles are for the two-spot model and crosses are for the one-spot model. The spots of the 1963 epoch are not plotted in Figure 31 because this epoch is far from most epochs and the spot parameters determined from only eight observations are less accurate compared to other epochs. Using equation (1), the relations between the spot longitude and time for V711 Tau were determined by the method of a least square analysis:

$$L1 = \underset{\text{S.D. } 9.10}{50.76}(t - 1975) + \underset{36.71}{326.58} \quad \text{--- (8)}$$

$$L2 = \underset{\text{S.D. } 9.39}{48.15}(t - 1975) + \underset{38.89}{254.37} \quad \text{--- (9)}$$

The top line was fitted to the open circles and expressed by equation (8). The bottom line was fitted to the filled circles and expressed by equation (9). The fit did not include the 1963 spots. The one-spot model (crosses) was not fitted. The starspot cycles of each spot for V711 Tau are determined as as 7.092 years and 7.476 years from equation (8) and (9) respectively.

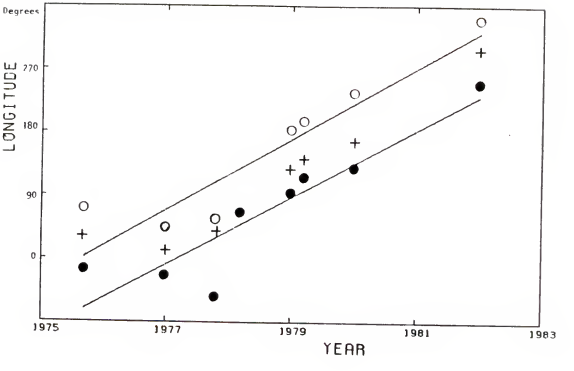

Figure 31: Spot Migration for V711 Tauri.

Conclusions

The twelve light curves of eight epochs for RS CVn, eleven light curves of six epochs for AR Lac, twelve light curves of nine epochs for V711 Tau, were analyzed and reproduced to investigate the cyclic variations of RS CVn binaries. The two-spot model was able to reproduce most light curves of the three systems.

The starspot cycles of RS CVn, AR Lac, and V711 Tau have been determined as a result of this dissertation. They are the first cycles inferred from the spot locations for each epoch. The starspot cycles of three systems for the two-spot model are tabulated in Table 54.

TABLE 54

Starspot Cycles of Three Systems.

Star	RS CVn	AR Lac	V711 Tau
Cycle of Spot 1	9.219 yrs	11.257 yrs	7.476 yrs
Cycle of Spot 2	8.998 yrs	15.180 yrs	7.092 yrs
Mean Cycle	9.109 yrs	13.218 yrs	7.284 yrs

Light curves of RS CVn taken between 1949 and 1979 were used for this investigation. In the above interval of 30 years, RS CVn has had the opportunity to complete three starspot cycles. Light curves were analyzed from 1972 to 1981 for AR Lac. However, because of completeness, only the

light curves between 1975 and 1981 were used to determine the starspot cycle of AR Lac. Thus the 13.2 year cycle of AR Lac had to be determined based on the light curves observed six years. Finally, for V711 Tau, light curves were analyzed from 1963 to 1981. Only the light curves between 1975 and 1981 were used to determine the 7.3 years cycle of V711 tau, as explained previously. Thus the 9.1 year cycle of RS CVn is the most confident result among the three systems. Though the 13.2 year cycle of AR Lac and the 7.3 year cycle of V711 Tau do not have the same confidence level as RS CVn, their cycles are not far off the 11 year sunspot cycle.

The mean spot temperatures of all epochs for each star is calculated and tabulated in Table 55. These mean temperatures in Table 55 are in agreement with a spot temperature of 3500 + 400 K derived by Eaton and Hall (1979) for the spotted RS CVn type star SZ Psc, and of 3400 + 100 K derived by Vogt (1981) for II Pegasi.

TABLE 55

Spot Mean Temperature of Three Systems.

Star	RS CVn	AR Lac	V711 Tau
Spot Temperature	3430 K	3600 K	3740 K
Star Temperature	4600 K	4550 K	4750 K
TF	0.746	0.792	0.788

Further research will be summarized as follows:

1. To apply our starspot model for other RS CVn binaries whenever the light curves are accumulated to analyze for the cyclic variation of RS CVn type stars.
2. To continue high precision photometry, preferably at R and I as well as UBV, to determine the physical properties of the spots better.
3. To study dynamo action within the outer convection zone which produces starspot magnetic fields.

LIST OF REFERENCES

Aller, H.D., Rugars, T.J., and Hodge, P.E., 1978, A.J., 83, 1935.

Antonopoulow, E., 1983, Astron. & Astrophys. Supple, 52., 381.

Bartolini, C., Blanco, C., Catalano, S., Cerruti-Solar, M., Eaton, J.A., Guarnieri, A., Hall, D.S., Henry, G.W., Hopkins, J.L., Landis, H.J., Leuth, H., Marilli, E., Piccioni, A., Renner, T.R., Rodono, M., and Scaltriti, F., 1983, Astron. & Astrophys., 117, 149.

Bartolini, C., Guarnieri, A., Piccioni, A., Catalano, S., Rodono, M., Brooke, A.F., Hall, D.S., Landis, H.J., Sarma, M.B.K., Olson, E.C., Renner, T.R., De Bernardi, C., and Scaltriti, F., 1978, A.J., 83, 1510.

Bidelman, W.P., 1954, Ap. J. Supple, 1, 175.

Blanco, C., Catalano, S., Marilli, E., and Rodono, M., 1981, IAU Int. Bull. Var. Stars No. 2000.

Blanco, C., Catalano, S., Marilli, E., and Rodono, M., 1982, Astron. & Astrophys., 106, 311.

Bopp, B.W., Espensk, F., Hall, D.S., Landis, H.J., Lovel, L.P., and Reucroft, S., 1977, A.J., 82, 47.

Bopp, B.W. & Evans, D.S., 1973, M.N.R.A.S., 164, 343.

Bopp, B.W., and Fekel, F., 1976, A.J., 81, 771.

Bopp, B.W., and Noah, P.V., 1980, Pub. A.S.P., 92, 717.

Bopp, B.W., and Talcott, J.C., 1978, A.J., 83, 1517.

Brown, R.L., and Crane, P.C., 1978, A.J., 83, 1504.

Budding, E., 1977, Astrophys. & Space Sci., 48, 207.

Catalano, S., Frisima A., and Rodono, M., 1980, Close Binary Stars: Obs. and Interpretation, ed. M.J. Plavec, D.M. Popper, and R.K. Ulrich, D. Reidel Publishing Co., Dordrecht, Holland.

Catalano, S., and Rodono, M., 1967, Mem. Soc. Ast. Ital., 38, 395.

----------, 1969, in IAU Colloquim No. 4, Non-Periodic Phenomena in Variable Stars, ed. L. Detre (Budapest: Academic Press) p. 435.

----------, 1974, Pub. A.S.P., 86, 390.

Caton, D.B., 1981, Ph.D. Dissertation, University of Florida.

Ceraski, L., 1914, A.N., 197, 256.

Chambliss, C.R., 1976, Pub. A.S.P., 88, 762.

Chambliss, C.R., Hall. D.S., Landis, H.J., Louth, H., Olsen, E.C., Renner, T.R., and Skillman, D.R., 1978, A.J., 83, 1514.

Chambliss, C.R., and Deterline, P.J., 1979, IAU Inf. Bull. Var. Stars No. 1591.

Chisari, D., and Lacona, B., 1965, Mem. Soc. Ats. Ital., 336, 463.

Cousins, A.W., 1963, M.N.A.S. South Africa, 22, 58.

Doiron, D.J., and Mutel, R.L., 1984, A.J., 89, 430.

Dorren, J.D., Siah, M.J., Guinan, E.F., and McCook, G.P., 1981, A.J., 86, 572.

Eaton, J.A., and Hall, D.S., 1979, Ap. J., 227, 907.

Eaton, J.A., Hall, D.S., and Henry, B.W., 1980, IAU Inf. Bull. Var. Stars No. 1862.

Eggen, O.J., 1966, Royal Obs. Bull. Var. Stars No. 120.

----------, 1978, IAU Inf. Bull. Var. Stars No. 1426.

Epstein, E.E., and Briggs, F.H., 1978, A.J., 83, 1487.

Ertan, A.Y., Tumer, O., Tunca, Z., Ibanoglu, C., Kurutac, M., and Evren, S., 1982, Astrophs. & Space Sci., 87, 225.

Evren, S., Ibanoglu, C., Tumer, Z., and Ertan, A.Y., 1983, Astrophs. & Space Sci., 95, 401.

Evren, S., Tunca, Z., Tumer, O., Ertan, A.Y., Ibanoglu, C., and Kurutac, M., 1980, IAU Inf. Bull. Var. Stars No. 1732.

Fekel, F.C., 1983, Ap. J., 268, 274.

Feldman, P.A., Taylor, A.R., Gregory, P.C., Seaquist, E.R., Balonck, T.J., and Cohen, N.I., 1978, A.J., 83, 1471.

Flin, P., and Sztajno, M., 1978, IAU Inf. Bull. Var. Stars No. 1394.

Fraquelli, D.A., 1978, A.J., 83, 1535.

----------, 1984, Ap. J., 276, 243.

Friedemann, C., and Gurtler, J., 1975, A.N., 296, 125.

Furenlid, I., and Young, A., 1978, A.J., 83, 1527.

Gainullin, A.S., 1943, Englehardt Bull., 22, 3.

Gibson. D.M., Hick, P.D., and Owen, F.N., 1978, A.J., 83, 1495.

Gibson, D.M., Hjellming, R.M., and Owen, F.N., 1975, Ap. J., 200, 199.

Gibson, D.M., and Hjellming, R.M., 1974, Pub. A.S.P., 86, 652.

Gibson, D.M., and Walter, F.M., 1982, Proc. of the South-West Regional Conf. for Ast. and Astrophs., Vol. 7, 21.

Gingerich, D., 1969, Theory and Observation of Normal Stellar Atmosphere, ed. C. Gingerich, The M.I.T. Press, Cambridge, Massachusetts, p. 453.

Gratton, L., 1950, Ap. J., 111, 31.

Guinan, E.F., McCook, G.P., Fraggda, J.L., and Weisenberger, A.B., 1979, IAU Inf. Bull. Var. Stars No. 1723.

Hall, D.S., 1972, Pub. A.S.P., 84, 323.

----------, 1975, Acta Astron., 25, 215.

----------, 1976, In IAU Colloquim No. 29, Multiple Periodic Variable Stars, ed., W.S. Fitch, Academic Press, Budapest, p. 287.

Hall, D.S., Richardson, T.R., and Chambliss, C.R., 1976, A.J., 81, 1138.

Harper, W.E., 1933, J.R.A.S. Canada, 27, 146.

Hearnshaw, J.B., 1978, A.J., 83, 1531.

Hiltner, W.A., 1947, Ap. J., 106, 481.

Hjellming, R.M., and Blankenship, L., 1973, Nature Phys. Sci., 243, 81.

Hobbs, R.W., Kondo, Y., and Feibelman, W.A., 1978, A.J., 83, 1525.

Hoffmeister, C., 1915, A.N., 200, 177.

----------, 1919, A.N., 208, 258.

Hoxie, D.T., 1973, Astron. & Astrphs., 25, 437.

Huenemoerder, D.P., and Ramsey, L.W., 1984, A.J., 89, 549.

Jacchia, L., 1929, Gaz. Astron., 16, 19.

Johnson, H.L., 1966, Ann. Rev. Astron. Astrophs., 4, 193.

Joy, A.H., 1930, Ap. J., 72, 41.

Keller, G., and Limber, D.N., 1951, Ap. J., 113. 637.

Kron, G.E., 1947, Pub. A.S.P., 59, 261.

----------, 1952, Ap. J., 115, 301.

Krzeminski, W., 1969, Low Luminosity Stars, ed. S. Kumar (London: Gordon & Breach), p. 57.

Kurutac, M., Ibanoglu, C., Tuncal, Z., and Ertan, A.Y., 1981, Astrophs. & Space Sci., 77, 325.

Landis, H.J., and Hall, D.S., 1976, IAU Inf. Bull. Var. Stars, No. 1113.

Landis. H.J., Lovell, L.P., Hall, D.S., Henry, G.W., and Renner, T.R., 1978, A.J., 83, 176.

Ludington, E.W., 1978, Ph.D. Dissetation, Univ. of Florida.

Maggini, M., 1916, Pub. Oss. Arcetri, Fasc. 34, 64.

Mohin, S., Raveendran, A.V., Mekkaden, M.V., Jayakumar, K., and Rosario, M.J., 1982, IAU Inf. Bull. Vra. Stars NO. 2190.

Mullan, D.J., 1974, Ap. J., 192, 149.

Mutel, R.L., and Weisberg, J.M., 1978, A.J., 83, 1499.

Naftilan, S.A., and Aikman, G.C.L., 1981, A.J., 86, 766.

Naftilan, S.A., and Drake, S.A., 1977, Ap. J., 216, 508.

Nations, H.L., and Ramsey, L.W., 1980, A.J., 85, 1086.

Nha, I.S., and Kang Y.W., 1982, Pub. A.S.P., 94, 496.

Nha, I.S., Park, H.S., Kang, Y.W., Wood, F.B., Williamson, R.M., and Leviton, D.B., in press Pub. A.S.P..

Oliver, J.P., 1974, Ph.D., Dissertation, UCLA.

----------, 1975, Pub. A.S.P., 87, 695.

Olson, E.C., 1982, IAU Inf. Bull. Var. Stars No. 2206.

Owen, F.N., and Gibson, D.M., 1978, A.J., 83, 1488.

Owen, F.N., Jones, T.W., and Gibson, D.M., 1976, Ap. J. Letters, 210, L27.

Owen, F.N., and Spangler, S.R., 1977, Ap. J., 217, L41.

Parthasarathy, M., Raveendran, A.V., and Mekkaden, M.V., 1981, Astrophys. & Space Sci., 74, 87.

Payne-Gaposchkin, C., 1939, Proceedings of American Philosophical Soc., 81, 189.

Plavec, M., 1967, B.A.C., 18, 334.

Popper, D.M., 1961, Ap. J., 133, 148.

----------, 1970, In IAU Colloquim No. 6, Mass Loss and Evolution in Close Binaries, ed. K. Gyldenkerne and R.M. West, Copenhagen Univ. Observatory, Copenhagen, p. 13.

----------, 1978, A.J., 83, 1522.

Ramsey, L.W., and Nations, H.L., 1980, Ap. J., 239, L212.

Rucinski, S.M., 1983, IAU Inf. Bull. Var. stars No. 2277.

Rugemer, H., 1931, A.N., 245, 39.

Russell, H.N., and Merrill, J.E., 1952, Princeton Univ. Obs. Contr., No. 26.

Sarma, M.B.K., and Ausekar, B.D., 1980, Acta Astron., 30, 101.

----------, 1981, Acta Astron., 31, 103.

Sanford, R.F., 1951, Ap. J., 113, 299.

Schneller, H., 1928, A.N., 233, 361.

Schneller, H., and Plaut, L., 1932, A.N., 245, 387.

Sitterly, B.W., 1930, Contr. Princeton Univ. Obs. No. 11.

Spangler, S.R., 1977, A.J., 82, 169.

Srivastava, R.K., 1981, Astrophys. & Space Sci., 78, 123.

Struve, O., 1946, Annales d'Astrophys., 9, 1.

Swank, J.H., White, N.E., Holt, S.S., and Becker, R.H., 1981, Ap. J., 246, 208.

Theokas, A.C., 1977, Astrophys. & Space Sci., 52, 213.

Torres, C.A.O., and Ferraz Mello, S., 1973, Astron. & Astrophys., 27, 231.

Vogt, S.S., 1981, Ap. J., 250, 327.

Walter, F.M., Charles, P., and Bowyer, S., 1978, A.J., 83, 1539.

Walter, F.M., Cash, W., Charles, P.A., and Bowyer, C.S., 1980, Ap. J., 236, 212.

Walter, F.M., Gibson, D.M., and Basri, G.S., 1983, Ap. J., 267, 665.

Weiler, E.J., 1977, IAU Cir. 3089.

----------, 1978, A.J., 83, 795.

Wilson, O.C., 1963, Ap. J., 138, 832.

Wilson, R.E., 1979, Ap. J., 234, 1054.

----------, 1983, privite communications.

Wilson, R.E., and Devinney, E.J., 1971, Ap. J., 166, 605.

Wood, D.B., 1971, A.J., 76, 701.

Wood, F.B., 1946, Priceton Univ. Obs. Contr. No. 21.

----------, 1979, Interacting Binary Star, p. 91.

Wyse, A.B., 1934, Lick Obs. Bull., 17, 37.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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